The plan of our study

Abstract A. Kawauchi suggests a formula assigning a point in the unit disk to a link (or a 3-manifold) and defines complex functions by using the points. We study the distribution of the points in the unit disk and properties of the complex functions. Since we have already enumerated the 444 links and 346 manifolds with lengths up to 10, we paint the disk with the points at the beginning.

Details Any link can be represented as closed braids and any 3-manifold can be realized as the 0-surgery along links in the three sphere. Since there is a one to one correspondence between the set of braid expressions and that of the finite sequences of integral numbers, we can define an injection from the set of links [resp. manifolds] to that of finite sequences of integral numbers. The finite sequences of integral numbers corresponding to links [resp. 3-manifolds] have the following form: If the number of the terms is 1, the sequence should be (0), and if the number of the terms is $n(> 1)$, the sequence $\mathbf{x}$ is of the form $\mathbf{x} = (1, x_2, \ldots, x_n)$ with $|x_i| \leq \frac{n}{2}$ $(i = 2, \ldots, n)$. We define a map from the set of links [resp. 3-manifolds] to the unit disk by assigning a point $w(\mathbf{x}) \in \mathbf{C}$ to the sequence $\mathbf{x}$ of the above form by the formulas:

$$r(\mathbf{x}) \underset{def}{=} \frac{|x_2|}{n^{n-1}} + \cdots + \frac{|x_n|}{n}, \quad \theta(\mathbf{x}) \underset{def}{=} \frac{1 - \frac{x_2}{|x_2|}}{2^n} + \cdots + \frac{1 - \frac{x_n}{|x_n|}}{2^2},$$

$$w(\mathbf{x}) \underset{def}{=} r(\mathbf{x}) \cdot \exp(2\pi\sqrt{-1} \cdot \theta(\mathbf{x})).$$

Furthermore, we define a complex power series f by

$$f(z) \underset{def}{=} \sum_{n=1}^{\infty} \prod_{\ell(\mathbf{x})=n} \frac{z}{1 - w(\mathbf{x})z},$$ where $\ell(\mathbf{x})$ is the number of the terms of $\mathbf{x}$, and $\mathbf{x}$ ranges over the finite sequences of integral numbers corresponding to the links [resp. the 3-manifolds].

We have the following problems: Where are the accumulation points for the set of points corresponding to the links [resp. the 3-manifolds]? What is the radius of convergence for the power series f?