Research Results

I am working on discretization of surfaces based on integrable systems techniques. In particular, I am interested in discretization of surfaces via Weierstrass-type representations. So far I have shown several results as follows:

1. **Research on semi-discrete minimal surfaces in Euclidean 3-space ([1])**

   Mueller, Wallner introduced semi-discrete isothermic surfaces in Euclidean 3-space and defined semi-discrete minimal surfaces. In a joint work with Wayne Rossman, we gave a Weierstrass representation for semi-discrete minimal surfaces and constructed new semi-discrete minimal surfaces. As an application, we discretized catenoid via various methods. In particular, we showed that semi-discrete minimal catenoids in the sense of Mueller, Wallner have the same profile curves as both discrete and smooth catenoids. This indicates that semi-discrete surfaces help us to recognize the similarities and differences between smooth and discrete surfaces.

2. **Research on discrete and semi-discrete maximal surfaces in Minkowski 3-space ([3], [4])**

   We discretized (and semi-discretized) maximal surfaces (spacelike surfaces with mean curvature identically 0) as a special class in isothermic surfaces in Minkowski 3-space. Unlike the case of smooth minimal surfaces in Euclidean 3-space, maximal surfaces generally have singularities, so it is natural to expect that discrete (or semi-discrete) maximal surfaces have singularities. We gave Weierstrass representations for discrete and semi-discrete maximal surfaces, and described their singularities. Singularities of discretized surfaces had not been explicitly defined before this. Based on these results, we aim to describe singularities of general discretized surfaces.

3. **Research on discrete linear Weingarten surfaces in hyperbolic and de Sitter 3-spaces ([5])**

   With Wayne Rossman, we analyzed the discrete linear Weingarten surfaces of Bryant type in hyperbolic 3-space introduced by Hoffmann, Rossman, Sasaki and Yoshida. Linear Weingarten surfaces of Bryant type are linear Weingarten surfaces which lie in a particular deformation family between flat surfaces and discrete constant mean curvature (for short, CMC) 1 surfaces. We considered singularities of discrete linear Weingarten surfaces of Bryant type and their unit normal vector fields in de Sitter 3-space. In particular, in a setting condition, we showed that the condition for singularities of discrete CMC 1 surfaces in de Sitter 3-space to appear is the same as the one for discrete maximal surfaces.

In addition, we gave smooth CMC surfaces with rotational symmetry via Weierstrass-type representations ([6]), and applying matrix factorizing theorems, we gave a Weierstrass-type representation for discrete CMC surfaces in Riemannian space forms ([7]).