Research results

My research interests focus on topology and geometry of spaces with torus actions and their relations with combinatorics. Below, I summarize my research results.

The cohomology rings of Hessenberg varieties ([1.5], [3.1] in the List)

[Joint work with Megumi Harada, Tatsuya Horiguchi, and Mikiya Masuda]

Hessenberg varieties are algebraic subsets of the flag varieites which provide a unifying description of such as Springer varieties, Peterson varieties, and toric varieties associated with root systems. In Lie type A_{n-1}, a Hessenberg variety is defined from an $n \times n$ matrix and a certain function $h : [n] \to [n]$. In particular, regular nilpotent Hessenberg variety $\mathrm{Hess}(N, h)$ and regular semisimple Hessenberg variety $\mathrm{Hess}(S, h)$ are well-studied classes. We first gave an explicit presentation of the cohomology ring $H^*(\mathrm{Hess}(N, h); \mathbb{Q})$. Also, by using the $\mathfrak{S}_n$-representation on $H^*(\mathrm{Hess}(S, h); \mathbb{Q})$, we obtained a ring isomorphism $H^*(\mathrm{Hess}(N, h); \mathbb{Q}) \cong H^*(\mathrm{Hess}(S, h); \mathbb{Q})^{\mathfrak{S}_n}$ providing a connection between different Hessenberg varieites.

The torus equivariant cohomology ring of the Springer variety ([3.3] in the List)

[Joint work with Tatsuya Horiguchi]

We gave an explicit presentation of the torus equivariant cohomology ring of the Springer variety associated to an arbitrary nilpotent matrix. We first constructed the torus equivariant analogue of Springer's representation of the symmetric group by using the localization technique of torus equivariant cohomology, and we showed that Tanisaki's argument for the ordinary cohomology of the Springer variety can be lifted for the equivariant cohomology.

The toric manifolds associated with root systems ([1.3] in the List)

Given a root system Φ, the collection of the Weyl chambers of Φ forms a fan, and we get a non-singular projective toric variety $X(\Phi)$. We provided a combinatorial rule to compute the intersection numbers for invariant divisors of $X(\Phi)$ by using Young diagrams. It is know that the cohomology ring of $X(\Phi)$ has a geometric basis, and as a corollary of this rule we provided a recursive formula for the structure constants with respect to the basis.

Pattern avoidance properties in Schubert geometry ([1.1] in the List)

Pattern avoidance is a combinatorial tool to characterize several geometric properties of Schubert varieties such as smoothness studied by Lakshmibai and Sandhya. This article is a survey on pattern avoidance properties in Schubert geometry.

Schubert calculus for weighted Grassmannians ([1.2] and [1.4] in the List)

[Joint work with Tomoo Matsumura]

Schubert calculus is a problem of computating the structure constants of the cohomology of the complex Grassmannian with respect to the Schubert classes. Topology, geometry, representation theory and combinatorics meet there via these structure constants. We introduced a natural definition of Schubert classes of the cohomology of the weighted Grassmannian orbifold, and computed the structure constants with respect to them. We also introduced an weighted analogue of Schur polynomials, and studied its relation to geometry and representation theory.

A generalization of convexity theorem of moment maps ([1.6] in the List)

For a Hamiltonian torus action on a compact symplectic manifold, it is know that the image of the moment map is a convex polytope. I gave a generalization of this theorem under a situation that the manifold admits three Hamiltonian torus actions, and discussed a relation to super-integrable systems arising in theoretical physics.