Research planning

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Hereafter we study the higher genus sigma function and Abelian function using the Miura canonical form, which gives defining equations of all algebraic curves. In particular we study the following problems.

1. Vanishing of sigma function for telescopic curves

Riemann's singularity theorem asserts that for a positive integer m determined from a geometric data of Riemann surface, any derivative of Riemann's theta function of degree less than m vanishes at e, which is an element of the theta divisor, and some derivatives of degree m does not vanish at e. However the theorem tells nothing on which derivatives do not vanish in general. In [5], by using the fact that the first term of the series expansion of sigma function is certain Schur function, from the properties of derivatives of Schur function, a derivative for which the sigma function for hyperelliptic curves does not vanish is given explicitly. This result extended to the curves $y^r = f(x)$ [1], (n, s) curves [3], telescopic curves (list of papers 1-2) and arbitrary Riemann surfaces [4]. Furthermore, in [1], by using the extended Jacobi inversion formulae, many derivatives for which the sigma function of the curves $y^r = f(x)$ does not vanish are given. In this research we show that the results on the vanishing of sigma function in [1] are satisfied for telescopic curves.

2. Sigma function of arbitrary Riemann surfaces

In [4], by using the fact that the Riemann's theta function associated with a Riemann surface is a solution of KP-hierarchy, it is shown that the first term of the series expansion of theta function is Schur function for arbitrary Riemann surfaces. Furthermore, from the properties of derivatives of Schur function, a derivative for which the theta function does not vanish is described in terms of gap sequence of a flat line bundle. For a hyperelliptic curve the gap sequence is described explicitly. In this research we consider to write the gap sequence and the derivative for which the theta function does not vanish explicitly for more general algebraic curves. In [4], by using the derivative for which the theta function does not vanish, the normalization constant for which the sigma function of arbitrary Riemann surfaces introduced in [2] is modular invariant is given. On the other hand the sigma function of (n, s) curves and telescopic curves has stronger algebraic properties than modular invariance, which the coefficients of the series expansion of the sigma function are polynomials of the coefficients of the defining equations. In this research we express the defining equations of arbitrary algebraic curves by Miura canonical form and show that the coefficients of the series expansion of the sigma funciton of arbitrary Riemann surfaces [4] are polynomials of the coefficients of the defining equations.

References

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