Previous research

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1. Study of sigma function for telescopic curves

Recently the multivariate sigma function introduced by F. Klein for hyperelliptic curves is generalized to the more general plane algebraic curves called (n, s) curves [1]. The sigma function is defined by modifying the Riemann's theta function in such a way that it does not depend on the choice of a canonical homology basis (modular invariant). Furthermore the sigma function has some remarkable algebraic properties. From these properties the sigma function has many applications in the problem of inversion of algebraic integrals and mathematical physics. In the theory of sigma funciton, a meromorphic bilinear form called algebraic bilinear form plays an important role. In [4] the algebraic bilinear form is constructed explicitly for (n, s) curves and it is shown that the first term of the series expansion of the sigma function of (n, s) curves is certain Schur function and the coefficients are polynomials of the coefficients of the defining equation. We constructed the algebraic bilinear form for telescopic curves [3], which contain the (n, s) curves, and showed that the series expansion of the sigma function of telescopic curves has the same algebraic properties as the (n, s) curves (list of papers 1-1). In [5] it is shown that the sigma function of (n, s) curves can be expressed by a tau function of KP-hierarchy. In [6], from the expression of the sigma function as a tau function, some properties of the series expansion and the vanishing of sigma function and the addition formulae of sigma function for (n, s) curves are derived. By extending the results, we showed the properties of the series expansion and the vanishing of sigma function for telescopic curves and derived the addition formulae of sigma function for telescopic curves, which is joint work with A. Nakayashiki (list of papers 1-2).

2. A generalization of Jacobi inversion formulae to telescopic curves

For a hyperelliptic curve of genus g it is known that k $(1 \leq k \leq g)$ points on the curve are expressed in terms of their Abel-Jacobi image by the hyperelliptic sigma function (Jacobi inversion formulae). In [2] the formulae are extended to the more general plane algebraic curves defined by $y^r = f(x)$. Furthermore in [2] a new vanishing property of sigma function of the curves $y^r = f(x)$ is derived by using the extended Jacobi inversion formulae. We extended the Jacobi inversion formulae to telescopic curves and showed that the vanishing property of sigma function in [2] is also satisfied for telescopic curves.

References

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