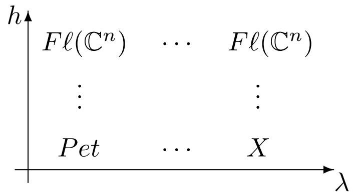
Research Plan

I have studied about nilpotent Hessenberg varieties as stated in Research Results. Recently, I am interested in regular Hessenberg varieties and its equivariant cohomology rings. Roughly speaking, they depend on a Young diagram λ and a Hessenberg function h. A regular Hessenberg variety is nilpotent when a Young diagram λ has exactly one row. A regular Hessenberg variety is semisimple when a Young diagram λ has exactly one column. On the other hand, a family of regular Hessenberg variety is a string between the Peterson variety Pet and the toric variety X whose fan is (Lie type A) Weyl chamber when a Hessenberg function is minimal in some sense. A family of regular Hessenberg variety is a family of flag varieties when a Hessenberg function is maximal. A family of regular Hessenberg varieties is shown in the following figure:



where the leftmost family is a family of regular nilpotent Hessenberg varieties and the rightmost family is a family of regular semisimple Hessenberg varieties. Also, the top family is a family of flag varieties. We already gave an explicit presentation of regular nilpotent Hessenberg varieties, and a relation between the cohomology rings of regular nilpotent Hessenberg varieties and the cohomology rings of regular semisimple Hessenberg varieties (List of Papers [1-1]). More precisely, letting $X_{\lambda,h}$ be a regular Hessenberg variety, we have the following isomorphism as rings

$$H^*(X_{\lambda=(n),h};\mathbb{Q}) \cong H^*(X_{\mu=(1^n),h};\mathbb{Q})^{S_n}$$

where an action of the symmetric group S_n on the cohomology of a regular semisimple Hessenberg variety is the dot action introduced by Tymoczko. We can conjecture the following isomorphism as rings between the cohomology rings of regular Hessenberg varieties and the cohomology rings of regular semisimple Hessenberg varieties (mentioned in List of Papers [1-1]):

$$H^*(X_{\lambda,h};\mathbb{Q}) \cong H^*(X_{\mu=(1^n),h};\mathbb{Q})^{S_\lambda} \tag{1}$$

where S_λ is the Young subgroup of S_n. Note that Brosnan-Chow already proved that both sides in (1) have the same Betti numbers. As the first step of the conjecture (1), we want to give an explicit presentation of the equivariant cohomology rings of the bottom family in the above figure which are strings between the Peterson variety Pet and the toric variety X whose fan is (Lie type A) Weyl chamber. From the presentation, we expect to give an explicit presentation of the equivariant cohomology rings of regular Hessenberg varieties and to prove the conjecture (1).