Research Results

I have studied nilpotent Hessenberg varieties which are subvarieties of (full) flag varieties. More precisely, we calculate equivariant cohomology rings of Springer varieties and regular nilpotent Hessenberg varieties which are the two extreme cases of nilpotent Hessenberg varieties. A maximal torus T of a flag variety naturally acts on the flag variety. It is known that an explicit presentation of the T-equivariant cohomology ring of a flag variety is given as equivariant version of Borel's presentation which is an explicit presentation of the cohomology ring of a flag variety. The torus T does not preserve nilpotent Hessenberg varieties in general, but the one-dimensional subtorus S of T introduced by Harada-Tymoczko preserves nilpotent Hessenberg varieties. We gave an explicit presentation of the S-equivariant cohomology rings of Springer varieties and regular nilpotent Hessenberg varieties. We state the research results as below.

1. Equivariant cohomology rings of Springer varieties (List of Papers [1-3], [2-1])
Springer varieties are associated with representations of a symmetric group which are called Springer's representations. In fact, the cohomology of Springer varieties is a representation of a symmetric group whose top degree cohomology is an irreducible representation. Moreover, every irreducible representation of a symmetric group can be constructed in such a way. An explicit presentation of the cohomology rings of Springer varieties is given by DeConcini-Procesi, and Tanisaki simplified their presentation. We gave an explicit presentation of the S-equivariant cohomology rings of the special Springer varieties (List of Papers [1-3]). However, we see that the S-equivariant cohomology of Springer varieties does not extend Springer's representation from the presentation. We gave an explicit presentation of the torus equivariant cohomology rings of Springer varieties such that the torus equivariant cohomology of Springer varieties does extend Springer's representation. This is joint work with Hiraku Abe (List of Papers [2-1]).

2. Equivariant cohomology rings of regular nilpotent Hessenberg varieties (List of Papers [1-1], [1-2])
Regular nilpotent Hessenberg varieties are generalization of Peterson varieties which are associated with the quantum cohomology of flag varieties. An explicit presentation of the S-equivariant cohomology rings of (Lie type A) Peterson varieties is given by Fukukawa-Harada-Masuda . We gave an explicit presentation of the S-equivariant cohomology rings of Peterson varieties in all Lie types which generalizes the presentation of Fukukawa-Harada-Masuda. This is joint work with Megumi Harada and Mikiya Masuda (List of Papers [1-2]). Furthermore, we gave an explicit presentation of the S-equivariant cohomology rings of (Lie type A) regular nilpotent Hessenberg varieties, and a relation between the cohomology rings of regular nilpotent Hessenberg varieties and the cohomology rings of regular semisimple Hessenberg varieties (List of Papers [1-1]).