Results

1. Hamiltonian minimality of normal bundles over the isoparametric submanifolds

   In 90's, Y.-G. Oh introduced the notion of *Hamiltonian-minimal* (*H-minimal*) Lagrangian submanifolds in Kähler manifolds. Such a submanifold is a critical point of the volume functional under the Hamiltonian deformation. This is an extension of the notion of *minimal* submanifold, and has been studied by many authors. An H-minimal Lagrangian submanifold is called *Hamiltonian-stable* (*H-stable*) if the second variation is non-negative for any Hamiltonian deformation. Oh studied H-stablity of some examples of H-minimal Lagrangian submanifold in a specific Kähler manifold. For example, the real projective space $\mathbb{R}P^n$ in $\mathbb{C}P^n$ and the standard tori in $\mathbb{C}^n$ are H-stable. On the other hand, we know a few family of H-minimal Lagrangian submanifolds in $\mathbb{C}^n$. We proved that any normal bundle of a principal orbit N of the adjoint representation of a compact semi-simple Lie group G in the Lie algebra $\mathfrak{g}$ is an H-minimal Lagrangian submanifold in the tangent space $T\mathfrak{g} \simeq \mathbb{C}^n$. These orbits are called the complex flag manifolds or C-spaces. Moreover, we characterize C-spaces by this property in the class of full and irreducible isoparametric submanifolds in $\mathbb{R}^n$.

2. On the minimality of normal bundles and austere submanifolds

   The normal bundle construction can be regarded as a generalization of the Gauss map of a submanifold. In fact, Palmer proves that the Gauss map of an isoparametric hypersurface in sphere is a minimal Lagrangian immersion into the oriented two-plane Grassmannian manifold $Gr_2(\mathbb{R}^{n+1})$.

   Generalizing this idea, we consider the minimality of normal bundles in the tangent bundle with the Sasaki metric. In particular, we investigate the mean curvature form of a normal bundle, and generalize some results.

3. On homogeneous Lagrangian submanifolds in complex hyperbolic spaces

   The classification of homogeneous submanifolds is an important problem in Riemannian geometry. In particular, there several classification results for homogeneous hypersurfaces. When the case of Lagrangian submanifods in a Kähler manifold, there are some results.

   In a joint work with T. Hashinaga, we consider homogeneous Lagrangian submanifolds in the complex hyperbolic spaces. In particular, we mention
   (1) constructions of homogeneous Lagrangian submanifolds by using a Kähler quotient,
   (2) a classification of homogeneous Lagrangian submanifolds obtained by a solvable Lie group action.
   As a consequence, we construct many examples of homogeneous Lagrangian submanifolds in $\mathbb{C}H^n$.

4. Stability of Legendre submanifolds in Sasaki manifolds

   There is a notion of *Sasaki manifolds*, which is an odd-dimensional counterpart to Kähler manifolds. In Sasaki manifolds, we consider *Legendrian-minimal* (*L-minimal*) Legendrian submanifolds which correspond to H-minimal Lagrangian manifolds in Kähler manifolds. We also define the notion of Legendrian stability for these submanifolds. The Riemannian cone of a Sasaki manifold is a Kähler cone, and the converse is true. When a Sasaki manifold is regular, it is a principal S^1-bundle of a Kähelr manifold. In these situations, the H-minimality of Lagrangian submanifolds corresponds to the L-minimality of Legendrian submanifolds by taking the cone or the projection. However, there are no correspondence between the H-stability and the L-stability. In fact, we prove that any closed L-minimal Legendrian submanifolds in the odd-dimensional unit sphere is L-unstable. In contrast to this situation, we give examples of L-stable closed curves in $SL(2, \mathbb{R})$ which is the Sasakian space form with constant ϕ-sectional curvature -7.