

The results of my studies (Shigeyasu KAMIYA)

- 1) The location of fixed points of elliptic elements of B-groups, which appear as boundary groups of Teichmüller space, was studied. If a B-group is a degenerate group, then one of fixed points is contained in the limit set. I showed that this elliptic fixed point is not located on the boundary of non-invariant component. And it is not a point of approximation. From this it follows that a degenerate group with elliptic elements is not geometrically finite.
- 2) I generalized the function theoretic properties of Fuchsian groups acting on the unit disc to those of discrete subgroups of $PU(1,n;\mathbb{C})$ acting on the complex unit ball, which is a model of complex hyperbolic space. I classified discrete subgroups $PU(1,n;\mathbb{C})$ into two types and studied their properties. In the case of convergence type, we can consider automorphic functions on the complex unit ball by the use of M-harmonic functions corresponding to Laplace-Beltrami operator. I also showed that a discrete subgroup of $PU(1,n;\mathbb{C})$ is convergence type if and only if the measure of the set of points of approximation is 0.
- 3) To investigate discrete subgroups of $PU(1,n;\mathbb{C})$, I studied basic properties of elements of groups. In particular, I extended Shimizu's Lemma in the real hyperbolic space to that in the complex hyperbolic space. Also I found some geometric invariants under $PU(1,n;\mathbb{C})$. This complex hyperbolic version and certain invariants were used to construct fundamental domains.
- 4) Jørgensen generalized Shimizu's lemma. I obtained several complex hyperbolic versions of Jørgensen's inequality. I also discussed the relation between our complex hyperbolic version of Jørgensen's inequality and a stable basin.
- 5) I considered the action on the product spaces of the boundaries of complex hyperbolic spaces to extend the results on Fuchsian groups by Tsuji.
- 6) A complex hyperbolic triangle group is a group generated by three complex reflections fixing complex geodesics in complex hyperbolic space. We can index a complex hyperbolic triangle group by a triple (p,q,r) , where $p,q,r \in \mathbb{N} \cup \{\infty\}$. For each triple, there is a one parameter family of complex hyperbolic triangle groups. Goldman and Parker studied ideal complex triangle groups and Schwartz showed a surprising relation between these groups and the complement of Whitehead link. This demonstrates the importance of this study. We restrict our attention to (n,n,∞) type. In particular, we have a list of non-discrete groups of type $(n,n,\infty;k)$. If n is greater than 21, then groups of type $(n,n,\infty;k)$ are not discrete. Also some discrete groups were found. The presentations for some lattices are studied.