In 2013, in a joint work with K.Ueda, we proved that Ebeling-Ploog's transpose duality extends to the polytope duality for families of weighted $K3$ hypersurfaces associated bimodular singularities and other isolated hypersurface singularities. Moreover, the polytope duality in this case may extend to the lattice mirror symmetry in the sense as follows:

Let Δ and Δ' be the reflexive polytopes obtained in the study of Mase-Ueda. Families $(\mathcal{F}_\Delta, \mathcal{F}_{\Delta'})$ of weighted $K3$ hypersurfaces associated to the polytopes Δ and Δ' are *lattice mirror symmetric* if an isometry of Picard lattices

$$\mathrm{Pic}(\Delta) \simeq U \oplus T(\Delta')$$

holds. In fact, among the isolated hypersurface singularities in question, in the following cases in the presenting list, the families attain lattice mirror symmetry:

Denote by $C_8^6 := \begin{pmatrix} -4 & 1 \\ 1 & -2 \end{pmatrix}$.

Singularity	Pic(Δ)	$\rho(\Delta)$	$\rho(\Delta^*)$	Pic(Δ^*)	Singularity
Q_{12}	$U \oplus E_6 \oplus E_8$	16	4	$U \oplus A_2$	E_{18}
$Z_{1,0}$	$U \oplus E_7 \oplus E_8$	17	3	$U \oplus A_1$	E_{19}
E_{20}	$U \oplus E_8^{\oplus 2}$	18	2	U	E_{20}
$Q_{2,0}$	$U \oplus A_6 \oplus E_8$	16	4	$U \oplus C_8^6$	Z_{17}
E_{25}	$U \oplus E_7 \oplus E_8$	17	3	$U \oplus A_1$	Z_{19}
Q_{18}	$U \oplus E_6 \oplus E_8$	16	4	$U \oplus A_2$	E_{30}

Now let us consider the following problem: let $(\Delta_{MU}, \Delta'_{MU})$ be a pair of reflexive polytopes obtained in Mase-Ueda's study, which does not attain the lattice mirror symmetry. Is it possible to take another pair (Δ, Δ') instead of $(\Delta_{MU}, \Delta'_{MU})$, such that the lattice mirror symmetry holds ?

Let Δ be a reflexive polytope with corresponding toric variety $\mathbb{P}_\Delta$ and $\tilde{\mathbb{P}}_\Delta$ denote its minimal resolution. For a generic anticanonical member Z of $\mathbb{P}_\Delta$, and its simultaneous minimal resolution $\tilde{Z}$, denote by $L_0(\Delta)$ the rank of the cokernel of a natural restriction

$$r : H^{1,1}(\tilde{\mathbb{P}}_\Delta) \to H^{1,1}(\tilde{Z}).$$

So far, we obtain the following negative answer for one case.

Example. Let us consider a self-dual transpose pair pair $B = B' = W_{18}$-singularity, and take a polytope

$$\Delta = \mathrm{Conv}\,\{(0,-1,0),\,(-2,3,0),\,(-3,5,-1),\,(1,-1,0),\,(0,0,1)\}\,.$$

Then, Δ is reflexive and $L_0(\Delta) = 0$.

No reflexive subpolytope of $\Delta_{[MU]} = \Delta_{(a;d)} = \Delta_{(3,4,7,14;28)}$ other than this satisfies $L_0 = 0$.
Indeed, starting from $\Delta_{[MU]}$ and we know that $\Delta_{[MU]}$ has an inner lattice point on the edge connecting $(0,-1,0)$ and $(2,-1,0)$ which makes L_0 grow by 6. So, we have to remove a vertex $(2,-1,0)$ from $\Delta_{[MU]}$. In order that to be reflexive, we have to remove a vertex $(-1,1,1)$ as well. The resulting subpolytope is the presenting Δ

We also have $\rho(\Delta) = 17$ and $\rho(\Delta^*) = 1$. This together with the fact that $L_0(\Delta) = 0$ leads that $\rho(\Delta) + \rho(\Delta^*) = 17 + 1 + 0 = 18 \neq 20$. Therefore, the isometry $\mathrm{Pic}(\Delta) \simeq U \oplus T(\Delta')$ does not hold. Thus for this pair, the answer seems NO.