

○Spacetimes with asymptotic Killing horizons

Recently we have investigated charged black string solutions residing in a five-dimensional Kasner universe [22]. Although there is no exact timelike Killing vector in the spacetime, the geometry is approximately static near the horizon. In fact, the spacetime admits a unique second order asymptotic Killing generator which satisfies an approximate Killing equation. To discuss physical properties of asymptotic Killing horizons, as a first step, we consider the confinement of the metric by assuming the existence of the n -th order asymptotic Killing generator in the spacetime. For example, in Einstein-Maxwell theory, if we specify the asymptotic structure and the physical quantities defined by the asymptotic Killing generator, we expect that the spacetime with the asymptotic Killing horizon is uniquely determined [23].

○Black hole solutions with extended electromagnetic fields

We focus on black holes coupled to the Born-Infeld nonlinear electrodynamics, the power-law Maxwell field, the exponential form of nonlinear electrodynamics, and the logarithmic form of nonlinear electrodynamics. Here, let us provide some motivations for considering such nonlinear electromagnetic fields. In the context of low-energy limit of heterotic string theory or as an effective action for the consideration of effects of loop corrections in quantum electrodynamics, considering strong electromagnetic fields in the regions near to pointlike charges, it was suggested that one may have to use generalized nonlinear Maxwell theory with quartic corrections of Maxwell field strength in those regions. Similar behavior may occur in the vicinity of compact objects and therefore it is reasonable to consider the nonlinear electrodynamics with an astrophysical motivation. Regarding these observations, it is worthwhile to study the effects of nonlinear electrodynamics on the geometrical behavior of black holes.

For example, like the Born-Infeld theory, a class of exponential form of nonlinear electrodynamics reduces to the linear Maxwell field in the special case. It is worth mentioning that although the exponential form and the logarithmic form of nonlinear electrodynamics do not cancel the divergence of the electric field at the origin, its singularity is much weaker than that in Einstein-Maxwell theory. This behavior is more natural with respect to the Born-Infeld theory in which the electric field of pointlike charges goes to a constant value. Then, motivated by the recent results mentioned above, we take into account the nonlinear electrodynamics to obtain four-dimensional and higher-dimensional charged rotating dilaton black hole solutions.

An exact charged rotating black hole solution in higher-dimensional Einstein-Maxwell theory has not been obtained. So we focus on the perturbative method to obtain such solutions as a first step. For example, in higher-dimensional Einstein-Born-Infeld-dilaton theory, we have constructed extremal charged rotating black hole solutions with equal angular momenta perturbatively [21]. Then we construct charged slowly/extremal rotating black hole solutions with nonlinear electromagnetic fields, the dilaton field, and a compact extra dimension. We investigate its geometries and the physical quantities such as the mass, the charge, the angular momentum, and the gyromagnetic ratio with nonlinear corrections. We also study the motion of test particles around the black hole and the existence of stable circular orbits. Then we discuss the verification of these black hole models by experiments and observations [24,25,26].