

The aim of confoliation theory is to unify topology of foliation and contact topology in dimension three. However, it has many difficulties even on its main subject called Thurston-Bennequin inequality. In contact topology the inequality characterizes tightness, namely, the absence of Lutz tubes. If a tight structure deforms to a foliation \mathcal{F} , the inequality also holds for \mathcal{F} . However the converse does not hold. On the other hand if a foliation does not contain a Reeb component, it satisfies the inequality. However the converse does not hold. (Indeed I constructed a foliation with Reeb component by deforming a given tight structure.)

Perhaps the difficulties are fatal in confoliation theory which is essentially a phenomenology. Similar difficulties appear as the difference between tightness and symplectic fillability. Although a weak fillable contact 3-manifold is tight, there are many tight contact manifold (e.g. one containing non-separating tori) which is not weakly fillable. On the other hand, Wand proved that a contact surgery by attaching a Weinstein 2-handle to the sub-level set of the symplectization preserves not only weak symplectic fillability but also tightness !

I would like to investigate the result of Wand and generalize it to high dimension. To this aim I introduced the following generalization of symplectic structure. We say that a 1-form α on an oriented $(2n+1)$ -manifold is a twisted contact structure with respect to a 2-form τ if $\alpha \wedge (d\alpha + \tau)^n > 0$. If α is also a twisted contact structure with respect to $\varepsilon\tau$ for any $\varepsilon \in (0, 1]$, we call it an $\varepsilon\tau$ -confoliation. Then we have $\alpha \wedge (d\alpha)^n \geq 0$ (Altschuler-Wu confoliation). Since a codimension-1 leaf-wise almost symplectic foliation is an example of $\varepsilon\tau$ -confoliation, it is natural to deform a contact structure to a Poisson structure through $\varepsilon\tau$ -confoliations. Indeed I obtained Mitsumatsu's Poisson structure from the standard contact structure on S^5 via $\varepsilon\tau$ -confoliations. We generalize symplectic structure by a similar twisting formulation. Then the natural filling condition of confoliation becomes a generalization of recently established weak symplectic fillability in high dimension. Using it I am trying to fill the gap between fillability and tightness.

I expect that my generalization of symplectic structure becomes the true even dimensional variation of contact structure and foliation. Even if it fails, I want to contribute another new object to topology from the above point of view.