

A closed 1-form on a manifold is locally derived from a function. In differential geometry, a tensor sends it to a vector field, and thereby associate to it a one-parameter group of diffeomorphisms, i.e., a flow. Namely, differential geometry bridges analysis to dynamical systems. Usually the tensor is either an inverse metric (gradient flow case) or a bivector field (Hamiltonian flow case). Riemannian and symplectic geometries are basic in these cases. We regard a bivector field as the pair of a Stefan singular distribution  $D$  and a 2-form  $\omega$  such that the restriction  $\omega|_D$  is non-degenerate. Then if  $D$  is the tangent space of a Stefan foliation, i.e.,  $D$  is integrable, and the restriction of  $\omega$  to each leaf is closed, the bivector field defines a Poisson structure. It is also symplectic when the manifold is a leaf.

More generally a Jacobi structure is defined by a vector field  $E$  (if it exists) such that i)  $E$  preserves the bivector field, ii)  $D$  and  $E$  spans the tangent space of a Stefan foliation, and iii) each leaf inherits a contact structure or a LCS structure. It can also be defined by a Lie bracket  $f, g$  supported in the intersection of the supports of functions  $f$  and  $g$  (Kirillov locality). The latter analytic definition well describes the universality of Jacobi structure than the former dynamical one.

Many author considered Jacobi structure as an analogue of Poisson structure. Indeed Jacobi manifold times  $\mathbb{R}$  is a Poisson manifold. However, from the above mentioned universality, Petalidou raised a modification problem of Jacobi structure into Poisson structure on the same manifold<sup>1</sup>. The importance of his problem has been recognized by experts through the result of Yoshihiko Mitsumatsu and my remark on it. Recently new results of younger people are appearing.

All of my results concerns the confoliation theory originally due to Eliashberg and Thurston. While the others investigated the relation between taut foliations and tight contact structures, I studied foliations with Reeb components and their generalization to high dimension. I obtained some results on convergence of contact structures into a foliation. Through my research, I have treated confoliation theory as the low dimensional Petalidou problem since a foliation by oriented surfaces is a Poisson structure. That is why I can relate the Mitsumatsu construction of Poisson structures with Reeb components on  $S^5$  to the Petalidou problem.

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<sup>1</sup>F. Petalidou. On a new relation between Jacobi and Poisson manifolds, J. Phys. A 35(2002).