I have studied on exact solutions for the higher-dimensional gravity to throw light on higher-dimensional black hole spacetimes in the context of higher-dimensional cosmological models and black hole formation at the LHC.

Uniqueness and non-uniqueness of the higher-dimensional black holes:

It is known that the Kerr solution is the unique black hole solution of the four-dimensional vacuum Einstein equations, whereas there exist various black hole solutions of the five-dimensional vacuum Einstein equations. For example, the Myers-Perry black hole solution with S^3 horizon and the Emparan-Reall black ring solution with $S^2 \times S^1$ horizon was known at that time. Hence we restricted ourselves to stationary black holes with spherical topology.

We showed the uniqueness of the asymptotically flat, black hole solution to the five-dimensional vacuum Einstein equation with a regular event horizon homeomorphic to S^3, admitting two commuting spacelike Killing vector fields and a stationary Killing vector field. The five-dimensional Myers-Perry black hole solution is unique in this class. [8]

In [16], we adapted the similar method to black rings. We showed that the only asymptotically flat black ring solution without a conical singularity to the five-dimensional vacuum Einstein equations is the Pomeransky-Sen'kov black ring solution.

Generating the exact solutions of higher-dimensional gravity:

It is difficult to solve the Einstein equations which are nonlinear partial differential equations. In some cases, one can generate new solutions from known ones.

One such technique is the inverse scattering method developed by Belinski and Zakharov. This method generates solitonic solutions from a known solution as a seed. In [13], we extended the inverse scattering method to the five-dimensional vacuum Einstein equations and showed the Mishima-Iguchi black ring solution with a rotating two sphere can be generated as two solitonic solution from five-dimensional Minkowski spacetime as a seed.

Under the assumption of the existence of two commuting Killing vector fields, the five-dimensional Einstein-Maxwell-Chern-Simons equations reduce to a nonlinear sigma model with the target space
$G_{2(2)} / (SL(2,R) \times SL(2,R))$. $G_{2(2)}$ isometry of the target space generates a charged solution of the five-dimensional Einstein-Maxwell-Chern-Simons equations from a vacuum solution of the Einstein equations. Applying the $G_{2(2)}$ generating technique to the Rasheed black hole solution, we generated a new rotating charged Kaluza-Klein black hole solution to the five-dimensional Einstein-Maxwell-Chern-Simons equations. [18]