Plan of Research

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Skein relation of Alexander Polynomial for handlebody-knots

We have the invariant $G_H^{(d)}$ for handlebody-knots comes from the Alexander polynomial which dose not depend on a choice of a meridian system.

These is a problem that the construction of a polynomial invariant which is defined by the skein relation like Jones polynomial in handlebody-knot theory. In knot theory, we can calculate the Alexander polynomial of knots using the skein relation. I think that we can calculate the Alexander polynomial of handlebody-knots using some skein relations as an analogy of knot theory.

In the paper [1], we have skein relations of the multivariable Alexander polynomial for colored links as previous research. We assume that the color of each component means a variable which is mapped to the meridian of the component. I think that we have the skein relation of the Alexander polynomial for handlebody-knots by adding "color" to each edge of the diagram of the handlebody-knot.

Minimal polynomial of Alexander ideal for handlebody-knots

We introduce the graph G_H as an invariant of handlebody-knot H which comes from the Alexander polynomial. There is infinite many handlebody-knots whose Alexander polynomial is trivial and whose Alexander ideal is non-trivial.

For example, Kinoshita's theta-curve satisfies this condition. The generator of the second Alexander ideal $E_2(s,t)$ of Kinoshita's theta-curve is not unique. However, the minimal polynomials of s and t in $E_2(s,t)$ are unique, respectively. For Kinoshita's theta-curve, the minimal polynomials of s and t in $E_2(s,t)$ do not depend on a choice of a meridian system, respectively. Thus, we have that Kinoshita's theta-curve is non-trivial. I would like consider the minimal polynomial the Alexander polynomial for other handlebody-knots.

References

[1] J. Murakami, A state model for the multi-variable Alexander polynomial, *Pacific J. Math.*, 157 (1993) no.1, 109–135.