Results of my research

Shin'ya Okazaki

A handlebody-knot is a handlebody embedded in the 3-sphere S^3, denoted by H. We obtained an invariant of handlebody-knots comes from the Alexander invariant. The Alexander polynomial is an invariant for a pair of a handlebody-knot and its meridian system. However, the invariant depend on only the handlebody-knot type.

For convenience, we suppose that the genus of a handlebody-knot is two. We choice a meridian system M for given handlebody-knot H. Let $\Delta^{(d)}_{(H,M)}(s,t) = \Sigma c s^x t^y$ be the d-th multi-variable Alexander polynomial for the pair (H, M). We have the graph from $\Delta^{(d)}_{(H,M)}(s,t)$ as follows.

For any Laurent polynomial $f(s,t) \in \mathbb{Z}[s^{\pm 1}, t^{\pm 1}]$, let $f(s,t) = \Sigma_i T_i = c_i s^x t^y$. For each term $T_i = c_i s^x t^y$ of $f(s,t)$, we take the black vertex b_i of the graph labeled by c_i. For any three terms $T_i = c_i s^{x_i} t^{y_i}$, $T_j = c_j s^{x_j} t^{y_j}$ and $T_k = c_k s^{x_k} t^{y_k}$, we take three points (x_i, y_i), (x_j, y_j) and (x_k, y_k) in $\mathbb{R}^2$, respectively. We take the white vertex of the graph labeled by

$$x_i y_j + x_j y_k + x_k y_i - x_i y_k - x_j y_i - x_k y_j$$

which is connected to three black vertex b_i, b_j and b_k. we have the graph, denoted by G_f. We define the vertex-weighted graph $G^{(d)}_H$ for (H, M) as $G_{\Delta^{(d)}_{(H,M)}(s,t)}$. Then the following theorem holds.

<u>Theorem 1</u>

The graph G_H up to multiplication by ± 1 to all labels of the black vertexes is an invariant of H.

As an application of Theorem 1, we have the following result. For any Laurent polynomial $f(s,t) \in \mathbb{Z}[s^{\pm 1}, t^{\pm 1}]$, we define the set $\hat{G}_f$ as follows. We calculate the factorization $f(s,t) = c f_1 f_2 \cdots f_n$, where $c \in \mathbb{Z}$ and $f_i \in \mathbb{Z}[s^{\pm 1}, t^{\pm 1}]$ is irreducible for $1 \leq i \leq n$. If $f(s,t) = 0$, then we define the set $\hat{G}_f$ as $\{G_f | f \in \mathbb{Z}[s^{\pm 1}, t^{\pm 1}]\}$, otherwise we define the set $\hat{G}_f$ as $\{G_{f_i} | 1 \leq i \leq n\}$. We define $\hat{G}^{(d)}_H$ as $\hat{G}_{\Delta^{(d)}_{(H,M)}(s,t)}$. Similarly, we define $\hat{G}^{(d)}_L$ as $\hat{G}_{\Delta^{(d)}_L(s,t)}$ for 2-component link. Then the following theorem holds.

<u>Theorem 2</u>

For any constituent link L of a genus handlebody-knot H, $\hat{G}^{(d+1)}_H \subset \hat{G}^{(d)}_L$.