RESEARCH RESULTS

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Group actions on manifolds give many examples of submanifolds as orbits. In particular, we can study orbits of linear isotropy representations and isotropy actions of compact symmetric spaces using the root system. Also, we can study orbits of Hermann actions which are a generalization of isotropy actions using the symmetric triad which is a generalization of notion of irreducible root system. The symmetric triad is introduced by O. Ikawa. We have studied the following research by using root systems and symmetric triads.

Area-minimizing cones Let S be the unit sphere in a Euclidean space $\mathbb{R}^n$, and let B be a submanifold of S. $C_B := \{tx \mid x \in B, 0 \leq t\}$ is called by a cone over B. In general, C_B has an isolated conical singularity at the origin. A cone C_B is called area-minimizing if $C_B^1 := \{tx \mid x \in B, 0 \leq t \leq 1\}$ has the least area among all integral currents with boundary B. If there exists area-nonincreasing retraction from $\mathbb{R}^n$ to C_B, then C_B is area-minimizing. We generalize the construction theorem of area-nonincreasing retraction which is showed by D. Hirohashi, T. Kanno, and H. Tasaki, and prove the following two theorems.
Theorem 1. Cones over minimal R-spaces which are isolated orbits of the linear isotropy representation of rank two symmetric spaces are area-minimizing except for seven exceptions.
Theorem 2. We can construct a new area-nonincreasing retraction using two area-nonincreasing retractions which satisfy some conditions.
This research is joint work with Takashi Sakai.

Weakly reflective submanifolds The notion of weakly reflective submanifolds which is introduced by O. Ikawa, T. Sakai and H. Tasaki is a generalization of the notion of reflective submanifolds. In the paper weakly reflective submanifold has been introduced, Weakly reflective orbits of the linear isotropy representation of irreducible symmetric spaces are classified. In this research, we construct weakly reflective orbits of commutative Hermann actions and group actions on Lie groups which are induced by commutative symmetric triads. Since weakly reflective submanifolds are austere, it is sufficient to show that weakly reflectivity of an austere submanifold. Austere submanifolds of commutative Hermann actions are classified by O. Ikawa. We classified austere orbits of actions on Lie group which induced by commutative compact symmetric triads using symmetric triads. Next, we described that a sufficient condition for austere orbits to be weakly reflective in terms of symmetric triads. Using this sufficient condition, we construct many examples of weakly reflective submanifolds in compact symmetric spaces and compact Lie groups.

Biharmonic submanifolds Research on harmonic maps has been made by many mathematicians. Harmonic maps are defined by critical points of the energy functional. When the map is an isometric immersion, a harmonic map is a minimal immersion. One of a natural generalization of harmonic maps is the notion of biharmonic map. A biharmonic map is a critical point of the bi-energy functional which is defined by the integral of the square of the length of the tension field. It is known that a harmonic map is biharmonic. Harmonic map and biharmonic map are characterized by some Euler-Lagrange equation. Thus, it is difficult to obtain a biharmonic map in general. However, by recent research of H. Urakawa, it was found that condition of biharmonic map is characterized by relation between the second fundamental form and the Ricci operator when the image of the map has the parallel mean curvature. Using this fact, we characterize the biharmonic condition of principal orbits of cohomogeneity one commutative Hermann actions in terms of symmetric triads. Using the method, we obtain new examples of biharmonic hypersurfaces in exceptional Riemannian symmetric spaces. This research is joint

work with Takashi Sakai and Hajime Urakawa.

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