Research results

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Let K be a knot and N a tubular neighborhood of K. For coprime integers $p(>0)$ and q, let $T(p,q)$ be the (p,q)-torus knot given on the standard solid torus V, which is homologous to $pl+qm$ in ∂V, where (m,l) is the standard meridian-longitude pair of V. Let φ: $V \to N$ be a faithful homeomorphism, that is, a homeomorphism sending the standard meridian-longitude pair of V to a meridian-longitude pair of K on N. Then we call the knot $\varphi(T(p,q))$ the (p,q)-cable knot, denoted by $K^{(p,q)}$, of K. For a knot invariant I, the function mapping K to $I(K^{(p,q)})$ is also a knot invariant, which is called the (p,q)-cable version of I. In general, it is expected that the (p,q)-cable version contains more information of knots than the original invariant has. For example, the cable version of the Jones polynomial, that is, the colored Jones polynomial is related to the volume conjecture. We focus on the Γ-polynomial, which is the common zeroth coefficient polynomial of the HOMFLYPT and Kauffman polynomials. We study the cable version of the Γ-polynomial.

• **On the braid index of Kanenobu knots**

Every knot is presented as a closed braid. The braid index of a knot is the minimum number of strings needed for the knot to be presented as a closed braid. The MFW inequality gives a lower bound of the braid index of a knot by applying the v-span of the HOMFLYPT polynomial. Since Kanenobu knots $k(n)$ $(n = 0, 1, 2, \dots)$ have the same HOMFLYPT polynomial, it is not easy to determine the braid index $\beta(k(n))$ of $k(n)$. As a result, we give a sharper lower bound of $\beta(k(n))$ by applying the $(2,q)$-cable version of the Γ-polynomial [2,3,4,7].

• **On the arc index of Kanenobu knots** (Joint work with Hwa Jeong Lee (KAIST))

Every knot has an arc presentation. The arc index of a knot is the minimum number of pages needed for the knot to be presented as an arc presentation. The MB inequality gives a lower bound of the arc index of a knot by applying the a-span of the Kauffman polynomial. Since Kanenobu knots $k(n)$ $(n = 0, 1, 2, \dots)$ have the same a-span of the Kauffman polynomials, it is not easy to determine the arc index $\alpha(k(n))$ of $k(n)$. As a result, we construct "canonical cabling algorithm" which gives sharper upper bounds of the arc indices of cable knots [10] and give a sharper lower bound of $\alpha(k(n))$ by applying "canonical cabling algorithm" and the $(2,q)$-cable version of the Γ-polynomial [9].

• **The cable Γ-polynomials of mutant knots**

A mutant knot is a possibly different knot obtained from a knot by an operation called mutation. It is known that many knot invariants are invariant under mutation, for example, the HOMFLYPT polynomial, the Kauffman polynomial, and their $(2,q)$-cable versions are invariant under mutation. On the other hand, it is known that the $(3,q)$-cable version of the HOMFLYPT polynomial distinguishes a mutant knot pair. As a result, we show that the $(3,q)$-cable version of the Γ-polynomial is invariant under mutation [1,6].

• **A characterization of the Γ-polynomials of knots with clasp number at most two**

Every knot bounds a singular disk with only clasp singularities, which is called a clasp disk. The clasp number of a knot is the minimum number of clasp singularities among all clasp disks of the knot. It is known that the Conway polynomials of knots with clasp number at most two are characterized. As a result, we characterize the Γ-polynomials of knots with clasp number at most two [5,8].