Research Plan

First, as a long-term target, I will study about Thomas-Yau conjecture which is one of important conjecture arising from the mirror symmetry and try to develop tools which may be used to understand the conjecture. The keywords of Thomas-Yau conjecture are special Lagrangian submanifolds, mean curvature flows and stabilities. As mentioned in Research Results, a Lagrangian mean curvature flow develops singularities in general, hence it does not have a long time solution in the ordinary smooth sense. Then Thomas introduce a notion of "stability" for a Lagrangian submanifold, and Thomas and Yau conjectured that if a Lagrangian submanifold L_0 is stable then the Lagrangian mean curvature flow L_t with initial condition L_0 exists for all time and converges to an unique special Lagrangian submanifold in its Hamiltonian deformation class.

Now I take notice of a conjecture that if the initial Lagrangian submanifold of a given Lagrangian mean curvature flow is "graded" then the mean curvature flow does not develop a singularity of type I in a Calabi-Yau manifold. This is very interesting since it can be considered as a necessity condition that Thomas-Yau conjecture is true. To my knowledge, this conjecture is still open though it is proved for some restrictive situations or some stronger assumptions. As a reason for difficulty, I pay attention to a technical confusion about two notions of "singularity" of a mean curvature flow. For a singularity of a mean curvature flow, there are notions of "special" singular points and "general" singular points, and one should treat these two notions carefully in some situations.

The first contribution to remove the above confusion is a paper of Stone. He proved that if the initial closed submanifold of a given mean curvature flow in $\mathbb{R}^{n+1}$ has codimension 1 and non-negative mean curvature and develops a singularity of type I then two notions of special and general singular points coincide. This result of Stone have been generalized to more general assumptions in $\mathbb{R}^N$. I expect that generalizing the result of Stone to general settings in a general Riemannian manifold helps technical detail in study about the above conjecture for graded Lagrangian submanifolds. Hence, as a middle-term target, I try to generalize the work of Stone to some general settings.

To generalize the result of Stone, as a short-time target, I work on generalizing Huisken's monotonicity formula to a general Riemannian manifold. Here Huisken's monotonicity formula is, roughly speaking, the integration of the backward heat kernel multiplied by an appropriate time dependent scaling terms on a submanifold moving along a mean curvature flow in a Euclidean space is monotone decreasing. However, when we do same procedure in a general Riemannian manifold, the corresponding value is not monotone decreasing because of effects of curvatures. Hence, I abandon the monotonicity of Huisken's monotonicity formula and generalize it in some weak sense, for example, the time derivative of the corresponding value can be positive but it is close to zero with some estimate.

After getting a suitable generalization of Huisken's monotonicity formula, I try to generalize the result of Stone in $\mathbb{R}^{n+1}$ to in a general Riemannian manifold. After getting a suitable generalization of the result of Stone, I attempt to prove the statement that if the initial Lagrangian submanifold of a given Lagrangian mean curvature flow is graded then the mean curvature flow does not develop a singularity of type I in a given Calabi-Yau manifold. When I find necessary tools to overcome some difficulty, I concentrate on developing those tools.