Research Plan

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Under my previous research of the transverse instability I study the following contents.

(1) Transverse instability of various equation

In my plan of the research, it is important to study the transverse instability on various equations. There is the relation between the difficulty of problems and treated standing waves to investigate the asymptotic behavior of solutions near by standing waves. Studying the stability and the linearized operator of standing waves on various equations, we find a suitable standing wave and a suitable equation to analyze the dynamics of solutions of the equation.

In the previous result, studying the behaver of solutions around the center manifolds precisely, Comech and Pelinovsky showed the instability of a standing wave with the degenerate linearized operator. Since the degeneracy of linearized operator in the our case is worse, we can not apply the argument by Comech and Pelinovsky to show the instability in the critical case. However, there is the possibility which we can show the instability in the critical case, if we combine between the analysis of the bifurcation and the argument. If we show the instability in the critical case by analyzing the behaver of solutions around the center manifolds, there is the possibility to show the behaver of solutions away from the unstable standing wave.

(2) The asymptotic stability of line standing waves

Under the suitable condition, we have known the asymptotic stability of the small grand state of a nonlinear Schrödinger equation (NLS) with linear potential on whole space $\mathbb{R}^N$. However, in the case of (NLS) on cylindrical space $\mathbb{R}\times\mathbb{T}_L$, it is difficult to prove the asymptotic stability because of the compactness of the transverse direction. Therefore, the proof of the asymptotic stability for (NLS) on $\mathbb{R}\times\mathbb{T}_L$ is difficult.

On the other hand, using a weighted norm in Zakharov–Kuznetsov equation (ZK), we can show the time decay of the scattering part of solution near by the line solitary wave of (ZK). In (ZK) with the weight which grows exponentially in the traveling direction, small solitary waves are not solutions with a finite value of the weighted norm. Therefore, I study the asymptotic stability of orbital stable line solitary wave of (ZK).

(3) The existence of the center stable manifolds

It is conjectured that solutions near by unstable solitary waves decompose some stable solitary waves and the the scattering part in many cases. This conjecture is true for some integrable equation, however it is very difficult to prove this conjecture for (ZK) which is not integrable. For the first step to approach solving this conjecture, I study the center stable manifolds near by unstable line solitary waves of (ZK) on $\mathbb{R}\times\mathbb{T}_L$.

To construct the center stable manifolds, the compactness of $\mathbb{T}_L$ and the existence of infinite many eigenvalues of linearized operator is the difficulty. To avoid the difficulty, I use a weighted norm. There are no small solitary wave with finite value of norm with the exponential type weight. From the absence of small solitary wave and the decay of the weighted norm of the scattering part there is the possibility to be able to construct the center stable manifolds.

(4) The asymptotic behavior of solutions away from unstable solitary waves

In some computer simulation, a solution with an initial data near the unstable line solitary wave splits into two solitary waves which are non-line solitary waves. I think solutions away from unstable line solitary waves decompose a multi-solitary wave and the scattering part. To show this conjecture, I study multi-solitary waves which are composed by different type solitary waves. Moreover, I investigate the relation among multi-solitary waves, the symmetry breaking bifurcation and the center stable manifolds to get the asymptotic behavior of solutions away from unstable line solitary waves.