Research Results

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I explain my results for the stability of line standing waves.

(i) Transverse instability for a system of nonlinear Schrödinger equations

I have studied the transverse instability for the line standing wave of the system of nonlinear Schrödinger equations (2NLS) on $\mathbb{R} \times \mathbb{T}_L$ which is simplified model of the Raman amplification in a plasma, where $\mathbb{T}_L = \mathbb{R}/2\pi L\mathbb{Z}$. A line standing wave is a standing wave of (2NLS) on $\mathbb{R} \times \mathbb{T}_L$ which is regarded a standing wave of (2NLS) on $\mathbb{R}$. If a standing wave on $\mathbb{R}$ is stable and is unstable as line standing wave on $\mathbb{R} \times \mathbb{T}_L$, we say that a line standing wave is transverse instability. In general, line standing waves are not the least energy solution of the stationary equation on $\mathbb{R} \times \mathbb{T}_L$. Therefore, it is difficult to prove the stability of line standing waves by only using the variational argument. Since line standing waves do not decay for the transverse direction, linearized operators do not has good conditions.

Since Rousset and Tzvetkov assume the smoothness of nonlinearities in the sense of the Fréchet differentiation in the previous results, we can not apply the argument of the previous results to (2NLS) which has a non-smooth nonlinearity. In the research (i), using the variational structure and the properties of solutions near by unstable standing waves, I have showed an estimate for the high frequency part of solution and obtained the stability of line standing waves for any non-critical period of $\mathbb{T}_L$.

(ii) Stability of line standing waves for nonlinear Schrödinger equations with critical period

In the previous results of transverse instability on $\mathbb{R} \times \mathbb{T}_L$, there exists the critical period L^* which is the boundary between the stability and the instability of line standing waves. In the case with the critical period, the line standing wave is a bifurcation point and the linearized operator around the line standing wave is degenerate. This degeneracy is different from the degeneracy treated by Comech and Pelinovsky and the linearized operator of the stationary equation is degenerate. So we can not apply the argument by Comech and Pelinovsky to the case with the critical period. Since the 2nd and 3rd order term of Lyapunov functional is degenerate, we can not use the result by Ohta.

In the research (ii), I have studied the stability of the line standing wave of nonlinear Schrödinger equation (NLS) with the nonlinearity $|u|^{p-1}u$ and the critical period. Analyzing the bifurcation of line standing wave, I have estimated the 4th order term of Lyapunov functional. By this estimate I could correct the degeneracy of the linearized operator and showed the stability of the line standing wave with the critical period for some exponents p.

Moreover, I have considered the stability of line standing waves for (NLS) with a linear potential and the nonlinearity $|u|^{p-1}u$. In the case of no linear potentials, since it is difficult to calculate precisely, I can not show the stability of the line standing wave with the critical period and some exponents p. In the case with a linear potential, we can treat small line standing waves. For sufficiently small line standing waves the interaction from the nonlinearity become weaker, so I can calculate the 4th order term of Lyapunov functional. Thus, for small line standing waves with the critical period I have obtained the critical exponent p^* which is the boundary between the stability and the instability.