Research plan

Building upon the results we obtained, we will consider the following problems:

1. **Construction of semi-discrete CMC surfaces in Riemannian space forms**

   Research on semi-discrete CMC surfaces in Euclidean 3-space had been started by Mueller, but he primarily focused on their theoretical aspects, so gave few examples of semi-discrete CMC surfaces. In the smooth and discrete cases, applying matrix factorizing theorems, we have Weierstrass-type representations for smooth and discrete CMC surfaces in Riemannian space forms ([9]). We will first determine matrix representations for semi-discrete CMC surfaces, then we will construct semi-discrete CMC surfaces, applying matrix factorizing theorems. In particular, from [1], we can expect that profile curves of semi-discrete CMC surfaces of revolution are the same as for both smooth and discrete ones. We will show these properties.

2. **Construction of discrete CMC surfaces in Minkowski 3-space**

   As an extension of the previous work in [9], we will describe discrete CMC surfaces in Minkowski 3-space. Unlike the case in Euclidean 3-space, because of the behavior of smooth surfaces, we can expect that discrete CMC surfaces in Minkowski 3-space generally have singularities. In [3], we defined singularities of discrete maximal surfaces, and we believe that this definition can be also applied to discrete CMC surfaces. In particular, we will construct three kinds of discrete CMC surfaces of revolution in Minkowski 3-space. If time permits, we will also try to describe discrete CMC surfaces in all Lorentzian space forms.

3. **Analyzing singularities of general discretized surfaces**

   Singularities of discrete surfaces had already been defined in [7], but, except for the case of discrete linear Weingarten surfaces with Weierstrass-type representations, it is much harder to analyze singularities of general discrete surfaces. The reason is that we cannot differentiate in the realm of discrete differential geometry. Furthermore, singularities of semi-discrete surfaces had been considered only in [6], [7], [11], so we have not yet defined singularities of general semi-discrete surfaces. It is hard to analyze singularities of general discretized surfaces, so we will first start by considering discretized constant Gaussian curvature surfaces, which do not have Weierstrass-type representations. Discretized constant positive Gaussian curvature surfaces can be obtained by taking parallel surfaces of discretized CMC surfaces, and discrete constant negative Gaussian curvature surfaces were already given by Schief. After we clarify singularities of discretized constant Gaussian curvature surfaces, we will characterize singularities of general discretized surfaces.