Research results

The geometry and topology of Hessenberg varieties ([1.1], [1.7], [2.1], [2.2] in the List)

• [Joint work with Megumi Harada, Tatsuya Horiguchi, and Mikiya Masuda] We gave an explicit presentation of the cohomology ring $H^*(\mathrm{Hess}(N,h);\mathbb{Q})$ of the regular nilpotent Hessenberg variety $\mathrm{Hess}(N,h)$, and by considering the $\mathfrak{S}_n$-representation on the cohomology ring of regular semisimple Hessenberg variety $\mathrm{Hess}(S,h)$, we obtained a ring isomorphism $H^*(\mathrm{Hess}(N,h);\mathbb{Q}) \cong H^*(\mathrm{Hess}(S,h);\mathbb{Q})^{\mathfrak{S}_n}$.

• [Joint work with Peter Crooks] The main interest of this project is the Hessenberg variety over the minimal nilpotent orbit. We described the Euler number, the irreducible components (type A), and a presentation of the cohomology ring. Also, for the natural torus action, we described its GKM graph.

• [Joint work with Lauren DeDieu, Federico Galetto, and Megumi Harada] We showed that the regular nilpotent Hessenberg variety $\mathrm{Hess}(N,h)$ in type A is a local complete intersection. We also studied a natural flat family of $\mathrm{Hess}(S,h)$ degenerating to $\mathrm{Hess}(N,h)$, and proved that their Hilbert polynomials are the same. As an application, we showed that the volume polynomial (computed by Abe-Horiguchi-Masuda-Murai-Sato) of the Poincaré duality algebra $H^*(\mathrm{Hess}(N,h);\mathbb{Q})$ gives the volume of the Newton-Okounkov body of $\mathrm{Hess}(N,h)$ under the Plücker embedding, and we determined this polytope for Peterson variety of type A_2 with respect to a choice of a valuation on the field of rational functions.

The torus equivariant cohomology ring of the Springer variety ([1.3] in the List)

[Joint work with Tatsuya Horiguchi] We gave an explicit presentation of the torus equivariant cohomology ring of the Springer variety. We first constructed the torus equivariant analogue of Springer's representation of the symmetric group by using the localization technique of torus equivariant cohomology, and showed that Tanisaki's argument for the ordinary cohomology of the Springer variety can be lifted for the equivariant cohomology.

The toric manifolds associated with root systems ([1.5] in the List)

Given a root system Φ, the collection of the Weyl chambers of Φ forms a fan, and we get a non-singular projective toric variety $X(\Phi)$. We provided a combinatorial rule to compute intersection numbers for invariant divisors of $X(\Phi)$ by using Young diagrams. It is known that the cohomology ring of $X(\Phi)$ has a geometric basis, and as a corollary of this rule we provided a recursive formula for the structure constants with respect to this basis.

Pattern avoidance properties and geometry of Schubert varieties ([1.2] in the List)

Pattern avoidance is a combinatorial tool to characterize several geometric properties of Schubert varieties such as smoothness studied by Lakshmibai and Sandhya. This article is a survey on pattern avoidance properties in Schubert geometry.

Schubert calculus for weighted Grassmannians ([1.4], [1.6] in the List)

[Joint work with Tomoo Matsumura] We introduced a natural definition of Schubert classes in the cohomology of the weighted Grassmannian orbifold, and computed the structure constants with respect to them. We also introduced a weighted analogue of Schur polynomials, and studied its relation to geometry and representation theory.