

# Previous research

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## 1. Study of sigma function for telescopic curves

Given the defining equations of algebraic curves, the sigma function is defined by the geometric data of the algebraic curve [1]. We consider a generalization of the sigma function by using Miura canonical form [3], which gives the defining equations for any algebraic curve. By extending the results of [4], we constructed a basis of the vector space of holomorphic one forms and a differential form called Fundamental differential of second kind for telescopic curves proposed in [3], which contain  $(n, s)$  curves (list of papers 1-2). By extending the sigma function of  $(n, s)$  curves to telescopic curves naturally, we find that the first term of the series expansion of the sigma function of telescopic curves at the origin is certain Schur function and the coefficients are polynomials of the coefficients of the defining equations. By extending the results of [5, 6], we obtained the relation between the sigma function of telescopic curves and the tau function of KP-hierarchy, the vanishing properties of the sigma function of telescopic curves, and the addition formulae for the sigma function of telescopic curves, which is joint work with A. Nakayashiki (list of papers 1-3).

## 2. A generalization of Jacobi inversion formulae to telescopic curves

For a hyperelliptic curve of genus  $g$ , it is well known that  $g$  points on the curve are expressed in terms of their Abel-Jacobi image by the hyperelliptic sigma function (Jacobi inversion formulae). In [2] the formulae are extended to the more general curves defined by  $y^r = f(x)$ . We extended the Jacobi inversion formulae to telescopic curves and showed that the vanishing property of the sigma function of telescopic curves (list of papers 1-1). From these results, we obtained the formulae which express a coordinate of one point on the curve from its Abel-Jacobi image.

## 3. A generator of the field of meromorphic functions on the sigma divisor of a hyperelliptic curve of genus 3

The quotient space obtained by dividing the set of zero points of the sigma function by the action of periods is called sigma divisor. We consider the field  $\mathcal{F}$  of meromorphic functions on the sigma divisor of a hyperelliptic curve  $V$  of genus 3. Let  $\mathcal{G}$  be the field of meromorphic functions on the 2-th symmetric products of  $V$ . By the Abel-Jacobi map,  $\mathcal{F}$  is isomorphic to  $\mathcal{G}$ . We derived a generator of  $\mathcal{G}$  and, by the Abel-Jacobi map, derived a generator of  $\mathcal{F}$  in terms of the sigma function, which is joint work with V. M. Buchstaber (list of papers 2-1).

## References

- [1] V.M. Buchstaber, V.Z.Enolski, D.V.Leykin, "Multi-Dimensional Sigma-Functions", arXiv:1208.0990.
- [2] S. Matsutani and E. Previato, "Jacobi inversion on strata of the Jacobian of the  $C_{rs}$  curve  $y^r = f(x)$ ", J. Math. Soc. Japan, Volume 60, Number 4 (2008), 1009-1044.
- [3] S. Miura, "Linear codes on affine algebraic curves", Trans. IEICE J81-A (1998), 1398-1421.
- [4] A. Nakayashiki, "On algebraic expressions of sigma functions for  $(n, s)$  curves", Asian J. Math. 14 (2010), 175-211.
- [5] A. Nakayashiki, "Sigma function as a tau function", IMRN 2010-3 (2010), 373-394.
- [6] A. Nakayashiki and K. Yori, "Derivatives of Schur, tau and sigma functions, on Abel-Jacobi images", in Symmetries, Integrable Systems and Representations, K.Iohara et al. eds., Springer, 2012, 429-462.