Research Plan

My research plan is to study the following problems.

- **The cohomology rings of regular nilpotent Hessneberg varieties**
  An explicit presentation of the cohomology rings of regular nilpotent Hessneberg varieties in types A, B, C, and G_2 are given (List of Papers [2-1], [2-2]). This result is obtained by using the fact of hyperplane arrangements since the logarithmic derivation module $D(\mathcal{A}_I)$ of an ideal arrangement $\mathcal{A}_I$ has been much studied. An explicit presentation of the cohomology rings of regular nilpotent Hessneberg varieties in types D, E(E_6, E_7, E_8), and F_4 are still open, so we will study this problem.

- **Research of regular Hessenberg varieties**
  We go toward the generalization of the following ring isomorphism:

  $$H^*(\mathrm{Hess}(N, I)) \cong H^*(\mathrm{Hess}(S, I))^W$$

  First we try to study this problem in type A. Let R_λ be regular (where λ is a size of Jordan block). Brosnan-Chow proved that the cohomology of the regular Hessenberg variety $\mathrm{Hess}(R_\lambda, h)$ is isomorphic to the cohomology of the regular semisimple Hessenberg variety $\mathrm{Hess}(S, h)$ as vector spaces:

  $$H^i(\mathrm{Hess}(R_\lambda, h)) \cong H^i(\mathrm{Hess}(S, h))^{\mathfrak{S}_\lambda}$$

  where if $\lambda = (\lambda_1, \ldots, \lambda_k)$, then $\mathfrak{S}_\lambda = \mathfrak{S}_{\lambda_1} \times \cdots \times \mathfrak{S}_{\lambda_k}$. We will study whether this isomorphism as vector spaces is isomorphism as rings.

- **The solution of Stanley-Stembridge conjecture**
  Shareshian-Wachs gave a significant step towards the solution of Stanley-Stembridge conjecture in graph theory. It means that this conjecture reduces to study the representation of a symmetric group $\mathfrak{S}_n$ on the cohomology $H^*(\mathrm{Hess}(S, h))$ of regular semisimple Hessenberg variety. So, we try to solve the Stanley-Stembridge conjecture from the connection between the regular Hessenberg variety $\mathrm{Hess}(R_\lambda, h)$ and the regular semisimple Hessenberg variety $\mathrm{Hess}(S, h)$.