Summary of Research

Finite type invariant for stable homeomorphism classes of curves on surface

To study so many link invariants, the concept of finite type invariant was born, which characterized the invariants by classifying as to common property. As similar to this concept, finite type invariant for stable homeomorphism classes of curves on surface was defined. In the original article [2], we defined finite type invariant for nanophrase which was a generalization of stable homeomorphism class of curves on surface. We then constructed the universal finite type invariant. Also, we identify the universal finite type invariant of degree 1 essentially with the linking matrix. For spherical curves, we showed that an extended Arnold basic invariants are finite type invariants of degree 2, but they do not provide a universal one of degree 2.

Extensions of Milnor's invariant to virtual links and handlebody-links under link-homotopy

A link-homotopy is an equivalence relation on links, which means the concept of "linking for components". Milnor defined a family of link-homotopy invariants, called Milnor's invariant. It is known that Milnor's invariant is a very strong invariant for link-homotopy. In the original article [1], we extended Milnor's invariant to virtual links, which are generalized links by adding virtual crossings to a link diagram. We also extended this invariant to welded links. In the original article [5], by joint working with Atsuhiko Mizusawa, we defined the concept of link-homotopy for handlebody-links, which is an embedding of finite handlebodys to the three sphere. We then extended Milnor's invariant to handlebody-links, by regarding a basis of first homology group of a handlebody-link with integer coefficient as a link. Furthermore we proved that any Milnor's invariant of a handlebody-link vanished if and only if the handlebody-link was link-homotopic to the trivial one. We also proved that when the lower degree Milnor's invariants vanished, the set of the link-homotopy class of handlebody-links was a one-to-one correspondence with the quotient space of tensor product space by the action of general linear group. Therefore we completely classified handlebody-links which had at most two components whose genus was more than one, by elementary divisors.

A relationship between Milnor's invariant and HOMFLYPT polynomial

Goussarov and Habiro introduced clasper theory, which gave a reconstruction of the theory of finite type invariant from the topological viewpoint. A clasper induced a local move on links. The local move is closely rerated to Milnor's invariant. By using this relation, Meilhan-Yasuhara showed that under some assumption some Milnor's invariant can be represented as a combination of HOMFLYPT polynomial of knots obtained by certain band sum of the link component. They also showed that their formula does not hold under weaker assumption. In the original article [3], by joint working with Akira Yasuhara, we improved their formula to hold the relation under weaker assumption. In particular, for any 4-component link the Milnor's invariants of length 4 are given by our formula. Further in the original article [4], we gave a similar relation between them for string links.