Summary of researches so far

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In the master thesis I generalized the theory of CW complexes to the spaces with action of a topological group G. About one year later Illman wrote his PhD thesis at Princeton University on the similar subject and became Professor at Helsinki University in 1975.

My PhD thesis showed that any topological manifolds of dimension > 4 is homeomorphic to a simplicial complex if and only if there exists a 3-dimensional homology sphere such that its Rokhlin invariant is non-zero and its double is zero in the homology cobordism group, using the newly proven theorem that the double suspension of homology sphere is homeomorphic to a sphere. About one year later this equivalence was proved in a little different way by Galewski-Stern and published in Annals of Math. Both results were published in Proc. Symp. Pure Math XXXII Part 2 and mine was treated as the first one. The existence problem of such a homology 3-sphere was negatively solved by C. Manolescu in 2013. So, by our result not only 4 dimension but also in dimension > 4 there are many topological manifolds which are not homeomorphic to any simplicial complex.

I continued various studies in topology. Among them the relation of Lusternik-Schnirelmann number and knots seems easy to understand and less popular. I used it several times as a theme of PhD thesis of student. The answer (usually -1) to the question how many contractible subsets are necessary to cover is called LS number. It was defined in 1933 as a topological invariant to solve some variation problems. An n-knot or simply knot is an embedded n-sphere in the (n+2)-space. Then I got the result that a locally flat knot is topologically trivial if and only if the complement has LS number 1. Moreover I constructed a knot whose complement has the given LS number and in another paper determined the fundamental group LS number of most Lie groups.

I have a result that although Riemannian geometry is the geometry of partial derivatives of rank < 3 of distance function on the diagonal, the information geometry is the geometry of its partial derivatives of rank < 4 on the diagonal. When I studied the metric of moduli space of instantons, I used a formula manipulation software to determine the sign of curvature. I had also PhD students studying computational geometry, mathematical physics or representation theory.

Recently I am busy to study the smooth unknotting conjecture for surface knot. But I translated a book of differential topology, studied a moduli space of Hopf spaces and had several results on the history of Japanese mathematics.