

## Plan of Research

Shin'ya Okazaki

### Litherland's Alexander polynomial for handlebody-knots

A handlebody-knot is a handlebody embedded in the 3-sphere. The Alexander polynomial is an invariant of a pair of handlebody-knot and its meridian system. Replacing a meridian system of a handlebody-knot acts on the Alexander invariant as an action of  $GL(g, \mathbb{Z})$ . In [2], we introduced an invariant  $G_H$  for handlebody-knots by using an invariant of the action of  $GL(g, \mathbb{Z})$  from the Alexander polynomial.

In [1], R. Litherland introduced the Alexander polynomial for  $\theta_n$ -curves. In general, the elementary ideal of the Alexander invariant is not principal for  $\theta_n$ -curves. Thus, there are infinitely many  $\theta_n$ -curves whose Alexander invariant is non-trivial and Alexander polynomial is trivial. However, the elementary ideal of the Litherland's Alexander invariant is principal, and the Litherland's Alexander polynomial is non-trivial for  $\theta_n$ -curve.

I would like to consider an action of the replacing a meridian system of a handlebody-knot for the Litherland's Alexander polynomial. I expect that an invariant of the action is stronger than  $G_H$ .

### Constituent handlebody-links for a handlebody-knot

We have an example that there is not the trivial knot which is a constituent knot of a handlebody-knot as previous research. In [1], R. Litherland gave a theorem which is stronger than Lemma 1 and Corollary 2 for  $\theta_n$ -curve by using the Litherland's Alexander polynomial. I would like to expand this theorem for constituent handlebody-links for a handlebody-knot.

## References

- [1] R. Litherland, The Alexander module of a knotted theta-curve, *Math. Proc. Camb. Phil. Soc.*, 106 (1989), 95–106.
- [2] S. Okazaki, An invariant coming from the Alexander polynomial for handlebody-knots, preprint.