Results of my research

Shin'ya Okazaki

A handlebody-knot is a handlebody embedded in the 3-sphere, denoted by H. The Alexander polynomial is an invariant for a pair of a handlebody-knot and its meridian system.

Let H be a genus g handlebody-knot and $D = \{D_1, D_2, \ldots, D_n\}$ a set of mutually disjoint essential disks properly embedded in H. Then $H(D) = \overline{H \setminus N(D)}$ is a handlebody-link in S^3. Here $N(D)$ is a regular neighborhood of D in H. We call $H(D)$ a *constituent handlebody-link* of H.

Let H be a genus g handlebody-knot, and $M = \{m_1, m_2, \ldots, m_g\}$ a meridian system of H. Let D_i be a meridian disk of H whose boundary is M_i for $1 \leq i \leq g$, and $D \subset \{D_1, D_2, \ldots, D_g\}$. Then $H(D)$ is a constituent handlebody-knot of H. Let $\Delta^{(d)}_{(H,M)}(t_1, t_2, \ldots, t_g)$ be the Alexander polynomial of (H, M).

Lemma 1 If $D = \{D_1\}$ and $M' = M \setminus \{m_1\}$, then

$$\Delta^{(d)}_{(H,M)}(1, t_2, \ldots, t_g) | \Delta^{(d)}_{(H(D),M')}(t_2, t_3, \ldots, t_g).$$

Corollary 2 If $D = \{D_1, D_2, \ldots, D_n\}$ and $M' = M \setminus \{m_1, m_2, \ldots, m_n\}$ for $n < g$, then

$$\Delta^{(d)}_{(H,M)}(1, 1, \ldots, 1, t_{n+1}, t_{n+2}, \ldots, t_g) | \Delta^{(d)}_{(H(D),M')}(t_{n+1}, t_{n+2}, \ldots, t_g).$$

We have the following theorem by Lemma 1.

Theorem 3 There is not the trivial knot which is a constituent knot of the following handlebody-knot.