Plan of Research

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The work of Alday-Gaiotto-Tachikawa in 2009 leads to a renewed attention to the connection between two-dimensional conformal field theories and four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories. The two-dimensional correlation function (conformal block) is identified with the instanton partition function of the gauge theory.

This "2d/4d connection" has been generalized to various directions. One of these generalizations is a correspondence between the "q-deformed" two-dimensional conformal field theories and the "q-lifted" five-dimensional supersymmetric gauge theories. The q-deformed 2d theories have q-Virasoro or q-W algebras, while for the q-lifted 5d theories, the 5-th direction is compactified on a circle S^1 whose radius is proportional to $\log q$.

In the 2d theory, the q-Virasoro algebra plays a central role. Its representation theory is still not well understood although 20 years have passed since its introduction. In particular, for a q-deformation of primary fields in the conformal field theory, the first principle definition of q-vertex operators is not known.

In our work, we showed that by taking the root of unity limit of q, the parafermion algebra can be obtained from the q-Virasoro or q-W algebras. We will also study this limit in more detail, and try to elucidate various properties of the 2d/4d connection. One of future theme of our study is the extension to the gauge theories on general ALE spaces or to quiver gauge theories. It is proposed an construction of associated "Yangian" for the quiver gauge theories. In the Schur-Weyl duality, the Yangian is connected with a Hecke algebra. Hence, an extension to these direction will deepen our understanding of the gauge/CFT/matrix model correspondence.

The "2d/5d connection" leads to various "2d/4d connections". Similarly, this "2d/5d connection" may be obtained from more fundamental connection. We will try to understand these connections from wider perspective.

Furthermore, it is known that the q-Virasoro and q-W algebras have close connection with a Hopf algebra, called the Ding-Iohara-Miki (DIM) algebra with a particular structure function. This structure function admits a generalization by introducing one more parameter p to obtain the so-called elliptic Ding-Iohara-Miki algebra. It is natural to expect that the "elliptic Virasoro algebra" and "elliptic W algebras" related with this elliptic DIM algebra play the crucial role in "2d/6d connection": the 2d correlation function with elliptic Virasoro/W symmetry is related with the partition function of 6d $\mathcal{N} = (2,0)$ superconformal theories. We will investigate this connection. If this "2d/6d connection" holds, we will also study roots of unity limit of parameters in these elliptic algebras.

Recently, we have studied the loop equations for the resolvent functions in the three-dimensional supersymmetric Chern-Simons-matter matrix models. We also continue this study. In the planar limit, the loop equations turn into two independent cubic algebraic equations. These loop equations are similar to those of ordinary two-matrix models. But the properties of the resolvent functions are quite different. In general cases, the form of the resolvent functions are not yet well understood. It is important to understand the resolvent functions in the Chern-Simons matter models.