Plan of the future studies

(1)   Virasoro algebra and the elliptic Calogero-Moser model

It is known that the trigonometric Calogero-Moser model has the symmetry of the Virasoro algebra. Further there exists an elliptic analog of the model, called the elliptic Calogero-Moser model. Thus we may guess that the elliptic Calogero-Moser model has connections to the free field realization of the Virasoro algebra on the torus. On the other hand, Langmann have studied the elliptic Calogero-Moser model by the free field realization. Therefore it would be possible to reformulate some results on the elliptic Calogero-Moser model based on the viewpoint of the free field realization of the Virasoro algebra on the torus.

(2)   An elliptic analog of the Macdonald polynomials

Since the trigonometric degeneration of the Ruijsenaars model can be solved due to the theory of the Macdonald polynomials, the studies of the Ruijsenaars model are directly connected with the studies of an elliptic analog of the Macdonald polynomials. Langmann has derived the functional equations of the kernel function for the elliptic Calogero-Moser Hamiltonian. From the functional equation, we can obtain the solutions to the elliptic Calogero-Moser model with p-derivative, where p is the elliptic modulus. On the other hand, the author have derived the functional equation of the kernel function for the Ruijsenaars operator by the free field realization. Then we notice that our functional equation is a q-analog of Langmann's one. Thus the functional equation of the kernel function for the Ruijsenaars operator contains important informations of an elliptic analog of the Macdonald polynomials.

(3)   Special eigenfunctions for the elliptic Ruijsenaars operator

There exists a kernel function for the elliptic Ruijsenaars operator called the dual Cauchy type kernel. The author has shown that an eigenfunction for the elliptic Ruijsenaars operator are obtained from the dual Cauchy type kernel function in a special case. It is expected that we can understand the special eigenfunctions based on the viewpoint of the representation theory of the elliptic Ding-Iohara-Miki algebra.