これまでの研究成果のまとめの英訳

佐藤　敬志

(1) background

Goresky, Kottwitz, and MacPherson [1] showed that one can obtain geometrical information of a manifold with a good torus action from the fixed points and one dimensional orbits. This method is called GKM theory. For example, we can determine the equivariant cohomology ring of flag manifolds combinatorially by GKM theory.

Let G be a simply-connected compact Lie group and T a maximal torus of G. The maximal torus T acts on the flag manifold G/T by left multiplication, and we can apply GKM theory to G/T with respect to this action of T. In this case, the fixed point set corresponds to the Weyl group of G, especially the cells of the Bruhat decomposition. This cell decomposition is made of the information of the Weyl group and the root system of G, so we can treat the equivariant cohomology ring combinatorially and geometrically.

(2) My results

When G is of classical type except for type C and of type G_2, the equivariant cohomology rings of the flag manifolds G/T have already been determined. I detemined the equivariant cohomology rings of the flag manifolds of type F_4 and E_6 explicitly as quotient rings of polynomial rings in a very different way from other reseachers. To determine them I introduced the notion of equivariant fiber bundles consisting of generalized flag manifolds, and then I proved the GKM version of the Leray-Hirsch theorem [2]. This theorem is stronger than the ordinary Leray-Hirsch theorem because this theorem determine also the ring structure using the combinatorics of Weyl groups. This theorem can reduce the complexity of determining the equivariant cohomology of the flag manifolds of exceptional types as possible, and this theorem shows that the equivariant fiber bundles are worth to be considered. Therefore the GKM Leray-Hirsch theorem is useful and theoretically important. I reconsider GKM theory and determine the equivariant cohomology ring of the flag manifold of type C with integer coefficients.

I obtained a great results on Hessenberg varieties as a joint work [3]. A Hessenberg variety is a subvariety of a flag variety determined by a “good” subset of the positive root system. On the other hand, a subset of the positive root system gives a hyperplane arrangement in Lie(T). A Hessenberg variety with some condition has a cell decomposition, and we found that the diminsions and the number of the cells are described in terms of the hyperplane arrangement. This is an generalization of flag varieties. The cohomology group of a such Hessenberg variety is already known, and then it is described in terms of the hyperplane arrangement. We found that not only as a group but also as a ring its cohomology is described in terms of the hyperplane arrangement. Even though the generators of its cohomology ring was not known, we showed that it is isomorphic to the quotient ring of $H^*(BT)$ by the ideal generated by a regular sequence.

References

[1] M. Goresky, R. Kottwitz, and R. MacPherson, Equivariant cohomology, Koszul duality and the localization theorem, *Invent. Math.* **131** (1998), no. 1, 25-83.

[2] T. Sato, The T-equivariant integral cohomology ring of E_6/T, arXiv:1406.3893.

[3] T. Abe, T. Horiguchi, M. Masuda, S. Murai, T. Sato, Hessenberg varieties and hyperplane arrangements, arXiv:1611.00269.