[Research Plan]

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[Research Project] Combinatorial characterization of international value vectors

(M,N)-Ricardo trade economy is a set $\mathcal{E} = \{A , \mathbf{q}\}$, where A is an $M \times N$ positive matrix and $\mathbf{q}$ is M-vector. International value vector $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ is normally defined as a couple of $\mathbf{w}$ and $\mathbf{p}$, in which $\mathbf{p}$ is perpendicular to a facet of the production possibility set defined by A and $\mathbf{q}$. However, these value vectors do not depend on $\mathbf{q}$. Then a program arises if we can characterize value vectors using only A and requiring some additional properties on value vectors.

A hopeful method is to use competitive types. Let $K_{M,N}$ be complete bipartite graph. Left vertices make a set of country indices and right vertices good indices. The competitive type with regards to $\mathbf{v}$ is defined as set of vertices (i,j) such that $w_i a_{ij} = p_j$. A competitive type is a subgraph of $K_{M,N}$. A spanning tree T defines a value vector $\mathbf{v}$ up to scalar multiplication that has competitive type T.

Two subgraphs of $K_{M,N}$ are in the same class when left indices are the same. When left indices sum up to $M+N-1$, the class is called spanning. A value vector $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ is called admissible when there exists a spanning tree T of $K_{M,N}$ such that $w_i a_{ij} = p_j \ \forall (i,j) \in T$ and $w_i a_{ij} \geqq p_j \ \forall (i,j) \in K_{M,N}$. By abuse of terms, such a spanning tree is also called admissible. A theorem tells that there is a unique admissible spanning tree in a spanning class, when A is in a general position. The set of all admissible spanning trees makes a tropical oriented matroid. A series of problems follows:

(1) Give the number of spanning trees in a given spanning class.

(2) When are two trees in a different class consistent?

(3) Build an algorithm which gives the unique admissible tree for a given class.

(4) Investigate flip relations between adjacent trees.

[Education project] Lecture series for common citizens

Tropical algebras such as min-plus semi-rings have become a powerful tool in the investigation of algebraic geometry, e.g. Maslov de-quantization. They are also used in applied mathematics such as time-event analysis. Min-times semi-ring plays a crucial role in the study of Ricardian trade theory and is easy to understand. International trade is a familiar topic in everyday economics. This combination can attract common people without any special training in mathematics. The fact that such an exotic algebra can be usefully employed in the research of international trade theory gives a good introduction to a feature of modern mathematics. Two or three lectures are planned.