Research Plan

A Lagrangian mean curvature flow develops singularities in general, hence it does not have a long time solution in the ordinary smooth sense. Then Thomas introduced a notion of "stability" for a Lagrangian submanifold, and Thomas and Yau conjectured that if a Lagrangian submanifold L_0 is stable then the Lagrangian mean curvature flow L_t with initial condition L_0 exists for all time and converges to an unique special Lagrangian submanifold in its Hamiltonian deformation class.

Now I pay attention to a statement that if the initial Lagrangian submanifold of a given Lagrangian mean curvature flow is "graded" then the mean curvature flow does not develop a singularity of type I in a Calabi-Yau manifold. This statement can be considered as a necessity condition that Thomas-Yau conjecture is true. To my knowledge, this conjecture is still open though it is proved for some restrictive situations or some stronger assumptions. There is a technical confusion about two notions of "singularity" of a mean curvature flow, "special" singular points and "general" singular points. Note that special is general by its definition. Rigorously speaking, there is so far a proof that "graded" implies no "special" type I singularity, and we do not know whether we can say that "graded" exclude "general" type I singularities or not.

A potential strategy to break down this problem is to show that every general type I singularity is actually a special type I singularity for mean curvature flows. Note that its converse statement is trivially true. The plausibility of this statement are the following two. First, for Ricci flows, it is true, that is, it is proved that every general type I singularity is actually a special type I singularity for Ricci flows. A key of its proof is "rigidity" of gradient shrinking Ricci solitons saying that a gradient shrinking Ricci soliton with vanishing scalar curvature at least one point is the Gaussian soliton. Second, Stone actually proved it for a codimension 1 mean curvature flow in $\mathbb{R}^{n+1}$ with non-negative mean curvature.

Important assumptions of the results due to Stone are the following three. (1) The ambient space is $\mathbb{R}^{n+1}$. (2) The codimension of the evolving submanifold is 1. (3) Its mean curvature is non-negative. A key of the proof of Stone's result is the classification of self-shrinkers with non-negative mean curvature due to Huisken. The proof itself is an argument by contradiction. If there exists a special type I singular point but not a general type I singular point, then as a rescaling limit we get a self-shrinker with $H \geq 0$ and vanishing second fundamental form at least one point but not a plane. However, there exists no such a self-shrinker by the classification due to Huisken.

It may be difficult to extend Huisken's classification for general codimension and without $H \geq 0$. If one could prove the "rigidity" of self-shrinkers saying a self-shrinker with vanishing second fundamental form at least one point is a plane, then the above problem, every general type I is actually a special type I ?, is proved affirmatively. Hence the problem that whether there is rigidity of self-shrinkers is important. However, the same argument as the proof of rigidity of gradient shrinking Ricci solitons does not work so far.

After Stone's work was published in 1994, Le and Sesum made a new contribution for the problem that whether every general type I is actually a special type I or not. One of their main statements says that every general type I singularity is actually a special type I singularity for mean curvature flows in a Euclidean space. They excluded assumptions (2) and (3) above. It seems that its proof include some technical difficulties however, its approach is very interesting. They used a pseudolocality theorem for mean curvature flows given by Chen and Yin in 2007. Originally, a pseudolocality theorem is proved by Perelman for Ricci flows, and Chen and Yin gave its mean curvature flow version. The proof of pseudolocality for mean curvature flows may also include some technical difficulties and chance of more generalization. Thus, in this period, I will try to follow technical detail of works due to Le-Sesum and Chen-Yin and to generalize them.