Research Results

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I explain my results for the stability of line standing waves.

(i) Transverse instability for a system of nonlinear Schrödinger equations

I have studied the transverse instability for the line standing wave of the system of nonlinear Schrödinger equations (2NLS) on $\mathbb{R} \times \mathbb{T}_L$, where $\mathbb{T}_L = \mathbb{R}/2\pi L\mathbb{Z}$. A line standing wave is a standing wave of (2NLS) on $\mathbb{R} \times \mathbb{T}_L$ which is regarded a standing wave of (2NLS) on $\mathbb{R}$. If a standing wave on $\mathbb{R}$ is stable and is unstable as line standing wave on $\mathbb{R} \times \mathbb{T}_L$, we say that this instability is the transverse instability. In general, line standing waves are not the least energy solution of the stationary equation on $\mathbb{R} \times \mathbb{T}_L$. Therefore, it is difficult to prove the stability of line standing waves by only using the variational argument. Since Rousset and Tzvetkov assume the smoothness of nonlinearities in the sense of the Fréchet differentiation on the previous results, we can not apply the argument of the previous results to (2NLS) which has a non-smooth nonlinearity. Moreover, on $\mathbb{R} \times \mathbb{T}_L$ it is difficult to get the orbital instability from the linear instability by the argument Georgiev–Ohata'12. In the research (i), using the variational structure and the properties of solutions near by unstable standing waves, I have showed an estimate for the high frequency part of solution and obtained the stability of line standing waves for any non-critical period of $\mathbb{T}_L$.

(ii) Stability of line standing waves for nonlinear Schrödinger equations with the critical period

For some line standing waves, there exists the critical period $L*$ such that if $L < L*$ then the line standing wave on $\mathbb{R} \times \mathbb{T}_L$ is stable and if $L > L^*$ then the line standing wave on $\mathbb{R} \times \mathbb{T}_L$ is unstable. For the critical period L^*, the linearized operator around the line standing wave on $\mathbb{R} \times \mathbb{T}_{L^*}$ is degenerate. From the degeneracy, we can not show the stability on $\mathbb{R} \times \mathbb{T}_{L^*}$ by applying the Vakhitov–Kolokolov condition for the stability of standing waves. Moreover, it is different from the case Comech–Pelinovsky'03 and Maeda'12 that the linearized operator of the stationary equation on $\mathbb{R} \times \mathbb{T}_{L^*}$ is also degenerate.

In the research (ii), I have studied the stability of line standing waves of the nonlinear Schrödineger equation with the nonlinearity $|u|^{p-1}u$ on $\mathbb{R} \times \mathbb{T}_{L^*}$. Constructing the estimate for 4th order term of a Lyapunov function, I recovered the degeneracy of the linearized operator. By using this estimate of the higher order nonlinearity, I showed the condition for the stability of standing waves on the bifurcation point of the stationary equation and proved the stability of the line standing wave with the critical period. Applying this condition, I proved the stability of line standing waves of the nonlinear Schrödinger equation with a linear potential.

(iii) Orbital and asymptotic stability for line solitary waves of Zakharov–Kuznetsov equation (ZK)

In this research, I have studied the orbital and asymptotic stability for line solitary waves of (ZK) on $\mathbb{R} \times \mathbb{T}_L$. Applying the argument (ii) for the critical period and the arguments of Rousset–Tzvetkov'09 and Pego–Weinstein'94, I proved the orbital stability and the instability of line solitary waves for any period L. To prove the asymptotic stability of solitary waves, it is important to apply the Martel–Merle argument using the Liouville type theorem. In the case with the critical period, the previous argument do not yields the Liouville type theorem from the degeneracy of the linearized operator. To recover the degeneracy, I used the difference between the solution and the modulated solitary waves. Constructing the corrected virial estimate for this difference, I proved the asymptotic stability of orbitally stable line solitary waves.