[Further research plan] The objective of this research is **that we make coloring invariants and quantum $U_q(\mathfrak{sl}_2)$ invariants stronger , categorize handlebody-knots and apply them to three-manifold invariants.** Until now, many invariants of handlebody-knots were separately defined and most of them were coloring invariants, and as for nontrivial quantum invariants, we only had quantum $U_q(\mathfrak{sl}_2)$ invariants. In addition, we little know about the strength and independence of each kind of invariants. In this situation, the applicant defines new invariants integrating these and aims for unified understanding. Such handlebody-knot invariants as the following figure 1 and the correlation between them indicate guidelines for a research. The usual knot theory has correlation similar to this figure (however, in case of the knot theory, there are no perturbative $\mathfrak{g}$ invariants and are Kontsevich invariants instead of universal perturbative invariants). We aim at the similar development in case of handlebody-knots to in case of the usual knot theory.

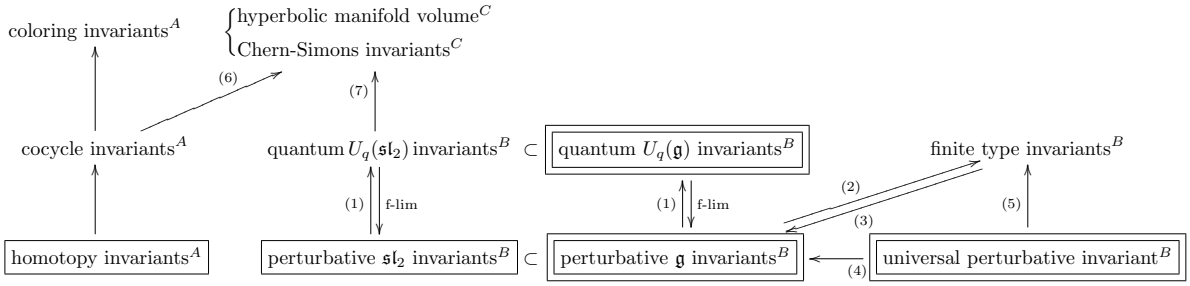

Figure 1: Relation between invariants

Here, invariants without a square are those of handlebody-knots which have already been defined. $\square$ indicates the invariants the applicant defined, and $\boxed{\square}$ does the invariants we will define and research on. Then, a superscript A represents coloring invariants with quandle. Other superscripts B and C represent quantum invariants and hyperbolic geometry respectively. In short, since these invariants have not yet been defined for handlebody-knots, we start with conjectured formulation on the relation of invariants. Moreover, an arrow in the figure shows that invariants at the head of the arrow can be gained from those at the tail: invariants at the tail of the arrow are stronger than those at the head. (1)$\sim$(7) of figure 1 show the applicant's conjectures (we infer that for handlebody-knots of genus 2, these conjectures will hold absolutely and for general handlebody-knots, these will hold with appropriate conditions). (1) Similarity with "Lawrence's conjecture," (2) Conjecture that coefficients are finite type invariants, (3) Conjecture that weight system will be defined, (4) Similarity with "LMO conjecture," (5) Conjecture that any finite type invariants can be gained, (6) Conjecture that cocycles which can lead to either hyperbolic volume or Chern-Simons invariants exist and (7) Similarity with "volume conjecture."