(i) Objects of the study

The goal of the study is to understand geometry of algebraic complex $K3$ surfaces by algebro-geometric method. We simply call an algebraic complex $K3$ surface a $K3$ surface.

It has been recently observed that $K3$ surfaces deeply relate with string theory, mirror symmetry, and conformal field theory in physics. In study of $K3$ surfaces in algebraic geometry, lattice theoretic methods are well-used due to Torelli-type theorem and results by [Nikulin]. Having many fruitful geometric properties, $K3$ surfaces are quite interesting objects of study.

We attack the following problems with new ideas from Lie algebra and Weierstrass semi-groups as well as lattice theory.

Problem 1. On moduli space of maps between a $K3$ surface and a Lie group.

Problem 2. On duality between families of $K3$ surfaces obtained from deformation of singularities.

Problem 3. On existence of projective models of Kummer surfaces whose Picard lattices are generated by lines.

(ii) Study methods

Problem1 By definition, an elliptic surface admits a holomorphic map to the projective line with general fibres being elliptic curves. Since the projective line is topologically equivalent to the 2-dimensional sphere, it can be regarded as a Lie group of matrices with determinant 1. Therefore, there naturally exists a map from an elliptic $K3$ surface to a Lie group. Being related to harmonic maps, it is important to study maps from a manifold to a Lie group in a prospect to differential geometry and differntial equations. We shall first characterise the moduli space of holomorphic maps from a $K3$ surface to a Grassmann variety, and then, we expand this to more general cases.

Problem 2 Important invariants of isolated hypersurface singularities in $\mathbb{C}^2$, which were classified by [Arnold] are modality and a-invariant.

It is shown by [Ebeling-Takahashi][Ebeling-Ploog] that there exists a combinatorical duality between invariants of hypersurface singularities with modality ≤ 2 or a-invariant ≤ 1 given by a transpose duality if they admit invertible polynomials. Moreover, [Mase-Ueda] and [Mase] concluded for certain transpose pairs that there also exists a lattice duality of families of $K3$ surfaces in the sense of Dolgachev.

We shall study a relation between the Milnor lattice of singularities and the Picard lattices of families of $K3$ surfaces. We expect a translation for this duality from the duality of homogology groups of families of $K3$ surfaces. Finally we intend to characterise the Frobenius structure of the base space of “unfolding” for singularities.

Problem 3 A Kummer surface is a $K3$ surface obtained by resolving singularities occured by the involution acting on a $(1, n)$-polarised abelian surface. We shall extend a result [Garbagnati-Sarti] for $n \leq 3$, by considering a problem whether or not there exists a nef divisor on a Kummer surface for $n \geq 4$ such that the intersection number with the divisor that gives the polarisation is 1, and the self-intersection numebr -2.

(iii) Aspects

Despite a deep study by [Aoki-Shioda] on algebraic cycles in $K3$ surfaces as nonsingular Fermat quartic, a lot of unknowns remain for other projective models. Problem 2 can have a proof of homological mirror symmetry that insists on an interchange of algebraic and vanishing cycles. We prospect the results of the study may be applied to the study of algebraic cycles on projective models of $K3$ surfaces.