

Plan of Study Yosuke Saito

Eigenfunctions for the elliptic Ruijsenaars operator
and the representation theory of the elliptic Ding-Iohara-Miki algebra

The elliptic Ruijsenaars system has three parameters q, t, p : $q, t \in \mathbb{C}^\times$ are q -deformation parameters, and $p \in \mathbb{C}$ is the elliptic modulus. If q, t and p are generic, it is very hard to solve the elliptic Ruijsenaars system. However, if q and t satisfy a special relation, we can obtain a special solution to the elliptic Ruijsenaars system.

First Komori-Noumi-Shiraishi showed the existence of the kernel function for the elliptic Ruijsenaars operator of dual Cauchy type. They also showed that if q and t satisfy a condition so-called the balancing condition, the kernel function of dual Cauchy type satisfies a functional equation. On the other hand, the author showed that the results due to Komori-Noumi-Shiraishi can be understood by the free field realization. In addition, the author showed that an eigenfunction for the elliptic Ruijsenaars operator is obtained from the functional equation of the kernel function of dual Cauchy type when q and t satisfy the balancing condition. The eigenfunction is regarded as an elliptic analog of elementary symmetric polynomials.

When the author derives the functional equation of the kernel function of dual Cauchy type by the free field realization, the kernel function is represented as a correlation function of boson operators. Then we can understand that the boson operators are intertwining operators of the elliptic Ding-Iohara-Miki algebra. Thus, it is expected that the eigenfunction for the elliptic Ruijsenaars operator obtained from the kernel function of dual Cauchy type is understood by the representation theory of the elliptic Ding-Iohara-Miki algebra. On the other hand, in 1995, Hasegawa showed that irreducible characters of the affine Lie algebra of type A are eigenfunctions for the elliptic Ruijsenaars operator when q and t satisfy the balancing condition. Therefore it is probable that the author's results have a connection with Hasegawa's study.