

Brief Summary of Study Yosuke Saito

The author has studied some materials which have connection to conformal field theory on the torus. Especially, the author intends to understand the elliptic Ruijsenaars system from the point of view of the representation theory of the elliptic Ding-Iohara-Miki algebra.

The Ruijsenaars system, which is an quantum many-body system, was introduced by Ruijsenaars as an q -deformation of the Calogero-Moser system. There exist three types of the Ruijsenaars system so-called rational, trigonometric, and elliptic type. In particular, the trigonometric Ruijsenaars system is well-understood by the theory of the Macdonald symmetric polynomials. On the other hand, in 2000s, it was known that a representation of a quantum group which is called the Ding-Iohara-Miki algebra was obtained from the free field realization of the Macdonald operator. Where the Macdonald operator is essentially the same as the Hamiltonian of the trigonometric Ruijsenaars system. Thus, we can understand that the trigonometric Ruijsenaars system has a connection with the Ding-Iohara-Miki algebra.

Then, the author had a question: how about in the elliptic case? That is, we may wonder whether there exists an elliptic quantum group which has a connection with the elliptic Ruijsenaars system. In the trigonometric case, a representation of the Ding-Iohara-Miki algebra is obtained from the free field realization of the Macdonald operator. Thus, in the elliptic case, it is a key point whether there exists the free field realization of the Hamiltonian of the elliptic Ruijsenaars system. In the following, we call the Hamiltonian the elliptic Ruijsenaars operator. In 2013, the author showed that the free field realization of the elliptic Ruijsenaars operator was derived from the kernel function of the operator which had been obtained by Ruijsenaars, and Komori-Noumi-Shiraishi. In addition, an elliptic analog of the Ding-Iohara-Miki algebra was obtained. We call the elliptic quantum group the elliptic Ding-Iohara-Miki algebra. Recently, gauge theory and topological string theory whose partition functions are represented by elliptic functions have been studied in physics. It is expected that the representation theory of the elliptic Ding-Iohara-Miki algebra has a connection with such materials in physics.