今後の研究計画の英訳

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Geometric structures of Hessenberg varieties are described well in terms of hyperplane arrangements. On the other hand, in GKM theory, the subspace of a full flag variety that consists of orbits of dimension less than 2 is also described in terms of the corresponding hyperplane arrangement. Therefore it is possible to unify my research on flag varieties and Hessenberg varieties and to understand them better.

(a) Pseudoreflection groups

Pseudoreflection groups are considered as generalized Weyl groups. The combinatorics of Weyl groups play an essential roll to investigate flag varieties by GKM theory, so it is natural to consider that there exist some spaces corresponding to pseudoreflection groups and their geometric structures are described well in terms of the combinatorics of pseudoreflection groups.

I will construct the "flag varieties" corresponding to pseudoreflection groups. I investigate the combinatorics of pseudoreflection groups and construct some cell complexes with the suitable cohomology rings from the combinatorics. A pseudoreflection group acts on $\mathbb{C}^n$, so the suitable cohomology ring means the quotient ring of the symmetric algebra on $\mathbb{C}^n$ by the invariant ideal.

On the Bruhat decomposition of a flag variety, each element of the Weyl group is given its length and it denotes the dimension of the cell. Surprisingly, there are very few results on the combinatorics of pseudoreflection groups which will corresponds to their geometric structures. I will grope for a method to assign an element of a pseudoreflection group its length and construct a corresponding cell complex from the hyperplane arrangement in $\mathbb{C}^n$.

(b) Hessenberg varieties

A regular nilpotent Hessenberg variety has a cell decomposition and the dimension of each cell is described in terms of the hyperplane arrangement. Moreover, not only as a group but also as a ring, its cohomology is determined by the hyperplane arrangement. However I have not understand yet geometrically the ideal of the quotient ring isomorphic to the cohomology ring. A Regular nilpotent Hessenberg variety has a good S^1-action and one can apply the localization theorem. The fixed point set can be identified with a subset of the Weyl group from the view point of the hyperplane arrangement. I will define the equivatiant "Schubert classes" corresponding to this cell decomposition.

(c) And more

I think that combinatorics and topology are very compatible because the GKM version of the Leray-Hirsch theorem is a very strong tool. In particular I will prove this theorem for some fiber bundle including the "flag varieties" corresponding to pseudoreflection groups.

I will unify two methods of hyperplane arrangements describing geometric structures on pseudoreflection groups and Hessenberg varieties.