

Research Plan

1 S-duality in the presence of the toric A-brane and new Hofstadter model

The correspondence that I studied in [3] is the consequence of the S-duality in the non-perturbative topological string theory without the toric A-brane. However, the S-duality should be maintained even in the presence of the toric A-brane from my previous study [5]. Therefore, I expect that the topological string theory with the toric A-branes describes a novel Hofstadter model. This project is important since I can study the S-duality through this correspondence. In order to see the S-dual structure, I try to obtain the Hofstadter butterfly in the non-perturbative topological string theory side.

2 Superconformal index of multiple M5-branes with the defect

It is suggested that the superconformal index of multiple M5 branes is given by integrating the partition function of M-string over the distance of the M5-branes. By using my previous studies [4, 6], I expect to define the superconformal index of multiple M5-branes with the codimension-2 defect. Since the integration is complicated, I try to integrate the partition function numerically in various values of the parameters. As an application of this result, I try to find a gravity dual theory which is predicted by the AdS/CFT correspondence.

3 Non-perturbative refined topological string theory

I plan to study the non-perturbative refined topological string theory. Based on my previous study [2], I expect that the two-parameter deformation of the Seiberg–Witten curves is related to the non-perturbative refined topological string theory. In order to consider the non-perturbative contribution, I use the fact that the quantization of the mirror curve is related to the non-perturbative topological string theory. First, I define the two-parameter deformation of the Seiberg–Witten curve in the operator formalism. Then, I calculate the expectation value of the operator. In this expression, I take the Nekrasov–Shatashvili limit in order to compare my result with the known results. This project is challenging, however, it enables to provide the non-perturbative definition of the B-model refined topological string theory.

References

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