Research Results

I am working on the classification of invariant Einstein metrics on generalized flag manifolds. Also, I am working on the classification of homogeneous geodesics in certain Riemannia spaces, for example generalized Wallach spaces, M-spaces and generalized C-spaces. So far I have obtain some results as following:

1. Research on invariant Einstein metrics on generalized flag manifolds([2], [5])

As we all known there are many interesting results about G-invariant Einstein metrics on generalized flag manifolds. Some meaningful work has been done mainly by A. Arvanitoyeorgos, I. Chrysikos and Y. Sakane. But when the number of the isotropy summands increases, it is very difficult to obtain the non-zero structure constants of the flag manifolds. We give a method which can be used to calculate structure constants of generalized flag manifolds with any number of isotropy summands.(c.f. [2], Theorem A). In this direction we present invariant Einstein metrics on some flag manifolds.

2. Research on classification of g.o.metrics in generalized Wallach spaces([3])

The classification of generalized Wallach spaces that was recently obtained by Yu.G. Nikoronov and Z. Chen, Y. Kang, K. Liang. They divide generalized Wallach spaces G/K into three types. In a joint work with Andreas Arvanitoyeorgos, we give the classification of g.o.metrics in generalized Wallach spaces G/K associate to the three types.

1) If $(G/K, g)$ is a space of type 1) then this is a g.o. space for any $\mathrm{Ad}(K)$-invariant Riemannian metric.

2) If $(G/K, g)$ is a space of type 2) or 3) then this is a g.o. space if and only if g is the standard metric.

3. Research on classification of g.o. metrics in M-spaces([4], [6])

We investigate homogeneous geodesics in a class of homogeneous spaces called M-spaces, which are defined as follows. Let G/K be a generalized flag manifold with $K = C(S) = S \times K_1$, where S is a torus in a compact simple Lie group G and K_1 is the semisimple part of K. Then the *associated M-space* is the homogeneous space G/K_1. These spaces were introduced and studied by H.C. Wang in 1954. We prove that for various classes of M-spaces the only g.o. metric is the standard metric. For other classes of M-spaces we give either necessary, or necessary and sufficient conditions, so that a G-invariant metric on G/K_1 is a g.o. metric. The analysis is based on properties of the isotropy representation $\mathfrak{m} = \mathfrak{m}_1 \oplus \cdots \oplus \mathfrak{m}_s$ of the flag manifold G/K (as $\mathrm{Ad}(K)$-modules).

In addition, we classified g.o.metrics in one class of homogeneous spaces called generalized C-spaces(c.f.[8]). Let G/K' be a generalized C-space determined by the generalized flag manifold G/K with $K_1 \subset K' \subset K$, $K = C(S) = S \times K_1$, S a torus in G, and K_1 the semisimple part of K. If $(G/K', g)$ is a g.o. space, then

$$g = \langle \cdot, \cdot \rangle = \Lambda \mid_{\mathfrak{s}_1} + \lambda B(\cdot, \cdot) \mid_{\mathfrak{m}}, \ (\lambda > 0),$$

where Λ is the operator associated to the metric g.