Research Plan

(1) (**Morse theory and GKM-theory**)

I continue to study on the existence of invariant Morse functions over GKM-manifolds.

Let a representation V be equivariantly embedded to an another representation W (the embedding is not necessary to be linear). Then, I consider whether one can construct a family of invariant functions over W such that each restriction $\Phi_\lambda|_V$ satisfies $\mathrm{Cr}(\Phi_\lambda|_V) = \{0_V\}$.

If such a family of invariant functions exists, it is seemingly possible to construct invariant Morse functions on GKM-manifolds by using the family and the result stated in the research plan.

(2) (**Freeness of graph equivariant cohomology**)

For an equivariantly formal GKM-manifold X, its equivariant cohomology $H_T^*(X)$ is known to be free over the polynomial ring $H_T^*(BT)$.

As a combinatorial version of this fact, we consider whether the graph equivariant cohomology $H_T^*(\mathcal{G})$ of a GKM-graph $\mathcal{G}$ is a free module over the polynomial ring. Wheres Guillemin-Zara already treated this problem when they introduced the notion of a graph equivariant cohomology, any satisfactory answer is still not known at this time (it is known that $H_T^*(\mathcal{G})$ is a free module over the polynomial ring under a certain Morse theoretic assumption).

This problem is important in its connection to the rigidity theorem and the following reconstruction problem:

(3) (**Reconstruction algorithm for GKM-graphs**)

As stated in the research result, GKM-graphs satisfy a certain rigidity with respect to graph equivariant cohomology algebras. Accordingly, as a refinement of this fact I hope to establish reconstruction algorithm for reconstructing a GKM-graph from its graph equivariant cohomology.

As a first step for this, I try to give a ring theoretic characterization of the notion of a 1-ideal.