Abstract of the results of my research

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The aim of representation theory of algebras is to investigate the structure of their module categories. So far I mainly have been studying the derived categories, the equivalences between them and some invariants under derived equivalences. They are useful tools in representation theory of algebras. For example, the important information of algebras such as the Grothendieck group and the finiteness of the global dimension is invariant under derived equivalences. Recently, it has turned out that they play important roles in Lie theory and noncommutative algebraic geometry.

The most important of various homological invariants in representation theory is the global dimension. The global dimension can be arbitrarily large. It thus reflects a minute property of the module categories.

Auslander experimentally proposed representation dimension, based on an idea that the representation-theoretic properties such as the representation-finiteness must be controlled by the homological invariants such as the global dimension. This concept was not handled easily. The representation dimension is, however, important because it has provided many interesting issues in representation theory of algebras and has become one of the origins of cluster tilting theory proposed recently.

An important concept introduced recently in connection with the representation dimension is Rouquier's dimension of triangulated category. This is an attempt to formulate in category theory the geometric 'dimension' as the global dimension is. The derived dimension (=the dimension of the derived category) and the stable dimension (=the dimension of the stable derived category) of algebras are closely related to the global dimension and the representation dimension (Rouquier) so that they are especially interesting in representation theory of algebras. Note that the derived dimension and the stable dimension are invariant under derived equivalences (Rickard).

Therefore, I considered that the representation-theoretic properties of algebras could be controlled by the derived dimension and the stable dimension. First, I would investigate the representation-theoretic properties of self-injective algebras having low stable dimension. By definition, if a self-injective algebra is representation-finite, then it has stable dimension 0. Then a natural question arises as to whether the converse should also hold. I gave an affirmative answer to this [2]. Although this was expected to hold by some experts, it had not been proved before. As an application, I exhibited in [4] (or [3]) that algebras having derived dimension 0 are closely related to self-injective ones having stable dimension 0, improving Chen-Ye-Zhang's result.

By the way, related to the study of self-injective algebras as mentioned above, I focused on Iwanaga-Gorenstein(=IG) algebras (see 'Research Plan') and their derived dimensions. It is, however, still a hard problem in general to give a precise value of the dimension of a given triangulated category. We hence introduce a concept of dimension of a triangulated category with respect to a fixed subcategory in [5] (This is joint work with Aihara, Araya, Iyama, and Takahashi). Then we proved in [5] that for an abelian category $\mathcal{A}$, an upper bound of the derived dimension of $\mathcal{A}$ with respect to a contravariantly finite subcategory $\mathcal{X}$ which generates $\mathcal{A}$ is given by the global dimension of the abelian category of finitely presented $\mathcal{X}$-modules. Our method not only recovers some known results on the derived dimensions in the sense of Rouquier, but also applies to various commutative and non-commutative noetherian rings. In fact, it is well-known that for a cotilting module T having injective dimension d over a noetherian ring Λ, $\mathcal{X}_T = \{X \in \operatorname{mod}\Lambda \mid \operatorname{Ext}^i_\Lambda(X,T) = 0 (\forall i > 0)\}$ is a contravariantly finite subcategory of the abelian category $\operatorname{mod}\Lambda$ of finitely generated Λ-modules and generates $\operatorname{mod}\Lambda$ (Auslander-Buchweitz). As a consequence of our result above, we obtained the fact that Λ has derived dimension with respect to $\mathcal{X}_T$ at most $d' = \max\{d, 1\}$.

Moreover, it turned out that if d is at least 1, then the derived dimension of Λ with respect to $\mathcal{X}_T$ is equal to $d' = d$ [8]. Applying this to a commutative noetherian Cohen-Macaulay(=CM) local ring R with a canonical module, we have the fact that the derived dimension of R with respect to the category $\mathrm{CM}(R)$ of CM modules is the Krull dimension of R. In addition, let Λ be an IG algebra having self-injective dimension d at least 1. Then Λ itself is a cotilting module having injective dimension d as a Λ-module. Hence, the derived dimension of Λ with respect to the category $\mathcal{X}_\Lambda = \mathrm{CM}(\Lambda)$ of CM modules is exactly d. Also, put $\Gamma = \begin{bmatrix} \Lambda & 0 \\ \Lambda & \Lambda \end{bmatrix}$. Then Γ is an IG algebra having self-injective dimension $d+1$. Hence, the derived dimension of Γ with respect to $\mathrm{CM}(\Gamma)$ is exactly $d+1$ and is strictly greater than that of Λ. This is an affirmative answer to Question A in 'Research Plan (2013)' (0).

Further, I gave an affirmative answer to the problem for a construction of IG algebras ('Research Plan' **(I)-1**) with Minamoto and Yamaura[10]. And I have started to study a classification of IG algebras ('Research Plan' **(I)-2,3**) with Iyama[11] and Enomoto.

On the other hand, as the duty of a CREST researcher, I developed a decomposition theory of modules by using Auslander-Reiten theory with Asashiba and Nakashima[6]. This work was done independently of the investigation due to Dowbor-Mróz.

In addition, I obtained the results related to a derived equivalence classification of self-injective algebras given by repetitive algebras with Asashiba, Kimura, and Nakashima (submitted [9]).