Sukuse Abe

[Our previous research]

[(1) On 4-cocycles of Alexander quandles on finite fields] Quandle shadow cocycle invariants are invariants of oriented surface-links. To calculate these invariants, we need concrete quandle 4-cocycles. For Alexander quandle X on the finite field, we represented a nontrivial 4-cocycle as a polynomial, and when this quandle is $H^2_Q(X;\mathbb{Z}) \cong 0$, we decided $H^4_Q(X;\mathbb{Z})$. This research has been developed by Nosaka Takefumi. In case of Alexander quandles on the finite field (generally regular quandles can be used.), by the universal coefficient theorem and the hurewicz fundamental homomorphism theorem, we gained $H^2_Q(X;\mathbb{Z}) \cong 0 \Rightarrow H^4_Q(X;\mathbb{Z}) \cong \pi_3(BX)$, and thus the generating elements of quandle homotopy invariants of surface links were decided.

[(2) Willerton conjecture] Generally, it is known that when for the normalized prime vassiliev invariant v_d of the degree d, the knot K has a diagram of n crossings, the value of $v_d(K)$ is shown by the order of n^d. Therefore, the set $\left\{ \left(\frac{v_2(K)}{n^2}, \frac{v_3(K)}{n^3} \right) \in \mathbb{R}^2 \middle| K \text{ has a knot diagram with } n \text{ crossings} \right\}$ is bounded. For some knots, points are plotted, and then a fish-link graph appears. We found what shape this graph could be for torus knots. Moreover, for these knots, we completely solved the problems Willerton conjecture posed.

[(3) Relation between quandle (shadow) cocycle invariants and finite type invariants] The relation between quandle (shadow) cocycle invariants and quantum invariants is not well known except the set the relation of the set-theoretic Yang-Baxter equation. We showed that a kind of special finite type invariants could be gained from quandle (shadow) cocycle invariants by giving filtration to knot determinant. Moreover, by applying this to three-manifold, we redefined finite type invariants different from Ohtsuki invariants for general three-manifold. It was shown that these invariants are calculable for a concrete lens space and for Brieskorn manifold, and are not empty sets. These invariants are stronger at least than Dijkgraaf-Witten invariants.

[(4) New quantum $U_q(\mathfrak{g})$ invariants of Special trivalent graphs] We gained the new quantum $U_q(\mathfrak{g})$ invariants and of Special trivalent graphs. These new invariants have the following characteristics. Although the two Special trivalent graphs to which a complement is homeomorphic cannot be distinguished by using usual quantum $U_q(\mathfrak{sl}_2)$ invariants. Those Special trivalent graphs can be distinguished by using newly defined quantum $U_q(\mathfrak{sl}_2)$ invariants. These invariants enable us to define the pertubative $\mathfrak{g}$ invariants and universal perturbative invariants and make it easy to calculate the pertubative $\mathfrak{sl}_2$ invariants. They can be applied to Representation theory and Prion graphs.

[(5) We obtain Vassiliev invariants of Quandle shadow cocycle invariants] We defined the formula of expansion that obtains Vassiliev invariants from Quandle shadow cocycle invariants. It was important that we consider the target set of Vassiliev invariants to be $\mathbb{Z}/p\mathbb{Z}$.