Previous research

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1. A generalization of Jacobi inversion formulae to telescopic curves

For a hyperelliptic curve of genus g, the coordinates of g points on the curve are expressed from their Abel-Jacobi image in terms of the hyperelliptic sigma functions (Jacobi inversion formulae). Matsutani and Previato [2] extended these formulae to the more general algebraic curves defined by $y^r = f(x)$, which are special cases of the (n, s) curves. In our research, we extended the sigma functions and the formulae of [2] to telescopic curves, which contain the (n, s) curves. More precisely, we expressed certain rational functions constructed by the coordinates of g points on a telescopic curve of genus g from their Abel-Jacobi image in terms of the sigma functions of the telescopic curve. Under a condition, we also gave similar formulae for k points on the telescopic curve of genus g, where $k < g$. Further, we gave some new vanishing properties of the sigma functions of telescopic curves by using the formulae.

2. Construction of the field of meromorphic functions on the sigma divisor of a hyperelliptic curve of genus 3

The zero set of a hyperelliptic sigma function in the Jacobian is called sigma divisor. In our research, for genus 3, we considered the field of meromorphic functions on $\mathbb{C}^3$ which are periodic on the sigma divisor, which is called the field of meromorphic functions on the sigma divisor. We derived a generating set of the field and all the relations of the elements of the generating set. Further, we showed that the field of rational functions on the symmetric square of a hyperelliptic curve of genus 3 is isomorphic to the field of meromorphic functions on the sigma divisor of the curve by the Abel-Jacobi map. As an application, we constructed solutions of the dynamical systems introduced in [1] for genus 3. These systems were constructed by Buchstaber and Mikhailov on the basis of commuting vector fields on the symmetric square of hyperelliptic curves. For genus 2, these systems correspond to the Dubrovin systems. This work is a joint work with V. M. Buchstaber.

3. Construction of two parametric deformation of the KdV-hierarchy

The hyperelliptic functions, which are defined by the logarithmic derivatives of the hyperelliptic sigma functions, are solutions of the KdV-hierarchy. In our research, for genus 3, we constructed two parametric deformation of the KdV-hierarchy by using the systems of [1]. This new system is integrated in terms of the hyperelliptic sigma functions of genus 3. When a hyperellptic curve of genus 3 degenerates into a hyperelliptic curve of genus 2, this new system becomes the KdV-hierarchy. Further, we derived a rational solution of the KdV-hierarchy by the rational limit of the solution of the new system. We showed that this rational solution is also obtained by the rational limit of the hyperelliptic functions of genus 2. This result would give an insight into the degenation of the sigma funcitons. When the coordinate x on the line tends to infinity, this rational solution tends to zero at a speed of $O(1/x^2)$. Therefore the Schrodinger equation whose potential is the rational solution has a meaningful solution from the point of view of physics. This work is a joint work with V. M. Buchstaber.

References

[1] V. M. Buchstaber, A. V. Mikhailov, "Infinite dimensional Lie algebras determined by the space of symmetric squares of hyperelliptic curves", *Functional Analysis and Its Applications*, Volume 51, Issue 1 (2017), 2–21.

[2] S. Matsutani, E. Previato, "Jacobi inversion on strata of the Jacobian of the C_{rs} curve $y^r = f(x)$", *Journal of the Mathematical Society of Japan*, Volume 60, Number 4 (2008), 1009–1044.