Research Plan

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My perspective of the research of this area is simple: construct various Lefschetz pencils, fibrations and symplectic 4-manifolds utilizing the techniques and knowhow that I have obtained and developed. I would like to understand symplectic 4-manifolds via those constructions.

With this motivation, I have several ongoing projects at present, each of which is highly promising since I have already got some relevant results. I am planning to focus on them for a while (a year or so).

1. *Small Lefschetz pencils and symplectic Calabi-Yau 4-manifolds.* This is a joint work with R. Inanc Baykur and Mustafa Korkmaz. We have already constructed a genus-2 smallest Lefschetz pencil, a genus-3 smallest hyperelliptic Lefschetz pencil and its generalization to higher genera. In addition, we have a lot of chances to apply them to construct Lefschetz pencils on symplectic Calabi-Yau 4-manifolds. We expect that one of those constructions leads to discovery of a new example.

2. *Small Lefschetz pencils on exotic rational surfaces.* This is another joint work with R. Inanc Baykur. Using the genus-2 smallest Lefschetz pencil in an effective and elaborated way, we have constructed an explicit genus-5 Lefschetz pencil on an exotic rational surface. This is interesting enough, but we are still eagerly looking for further small exotic rational surfaces by similar method.

3. *Lefschetz pencils on the complex projective surface* $\mathbb{CP}^2$. The complex projective surface $\mathbb{CP}^2$ is another fundamental example of a symplectic 4-manifold. As Kenta Hayano and I found satisfactorily many Lefschetz pencils on the four-torus, I am constructing Lefschetz pencils on $\mathbb{CP}^2$. Indeed, I have already covered "two-thirds" of all the possibilities.

4. *Sections of Gurtas's Lefschetz fibration, Okuda-Takamura's Lefschetz fibration and other interesting Lefschetz fibrations.* This is a joint work with Naoyuki Monden. Concerning (-1)-sections, there are a couple of important Lefschetz fibrations which should be studied. In particular, examples found by Gurtas and Okuda-Takamura are interesting. I have some reasonable clues to find sections of those Lefschetz fibrations.

5. *Lefschetz pencils and their unbranched finite coverings* Taking unbranched finite coverings of Lefschetz pencils provides new Lefschetz pencils. I have found an interesting relationship among the Matsumoto-Cadavid-Korkmaz Lefschetz fibrations regarding their coverings.

Future plan. I have developed techniques for constructing Lefschetz pencils and I am sure I will learn more from the ongoing projects. Once I get sufficiently enough skills, it would be possible to tackle more ambitious problems. One of such problems is the existence problem of a symplectic 4-manifold that violates the *Bogomolov-Miyaoka-Yau inequality*. This inequality holds for a most dominant class of complex surfaces, i.e., *of general type*. Thus this problem asks the difference of the complex surfaces and the symplectic 4-manifolds. It has been a couple of decades since this problem first appeared, but no one knows the answer. As a future project, I would like to try to prove the existence by explicitly constructing Lefschetz pencils.