Works

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A *link* is the union of mutually disjoint, finite number of oriented 1-sphere(s) in the 3-sphere S^3. In particular, one component link is called a *knot*. In general, knots and links are drawn in the plane (, more generally, for surfaces) as *link diagram*. More precisely, for a link in the 3-dimensional space or the 3-sphere, the image of projection of it contains multiple points. An image is said to be a *projection* (or *spherical curve*), if the set of the multiple points consists of finitely many transverse double points. It is one of the standard styles for studying the structure of knots and links, to apply local transformation on link diagram (we say that such local transformation an *operation*).
I' d like to consider the following problem;
Characterize equivalence classes by an equivalence relation induced by an operation for link diagrams

It is difficult to solve. Then I treat projections or spherical curves.
Ito-Takimura[IT1,2] and I-T-Taniyama[ITT] characterized some equivalence classes induced by *Reidemeister moves on spherical curves*.
In 2010, Kengo Kishimoto introduced an operation called *region crossing change*. Then Ayaka Shimizu proved that r.c.c. is an unknotting operation in [S]. In fact, in [S], she showed that any set of crossings of each knot diagram can be changed by a r.c.c. Further in general, it is remarked that for a link diagram *D*, a link diagram of the trivial link cannot be obtained from *D* by any r.c.c. Further more, Kawauchi-Kishimoto-Shimizu proposed a game based on region crossing change called "Region Select". Recently Ayumu Inoue-Ryo Shimizu [IS] introduced another operation called *region freeze crossing change* related to region crossing change. They showed that there exists a knot diagram such that some changes of crossings cannot be realized by r.f.c.c.

1. We defined a complex for spherical curves.
   A 0-simplex corresponds to an equivalence class by equivalence relation induced by operation of Reidemeister move I, say RI. There is an edge between two vertices *v* and *v'* of this complex if and only if a spherical curve corresponding to *v'* is obtained from a spherical curve corresponding to *v* by some RI's and a single weak RIII.

In [FHIKM], we decided the complex for spherical curves under 7 double points (2017).
2. In [HI], we give a necessary and sufficient condition: For a spherical curve *P*, spherical curve *P'* is obtained from *P* by deformations of type RI and RIII (2018).
3. For spherical curves *P* and *P'*, suppose that *P'* obtained from *P* by deformations of type RI and RIII. Then we estimated the minimal number of strong RIII from *P* to *P'* (2018).
4. In [H4], I defined a degree of difficulty of Region Select and composed the most difficult game (2017).
5. We induced a $\mathbf{Z}_2$-linear map Φ from the set based on the regions to the set based on the crossings defined by r.c.c. and made use it to study r.c.c. In fact, we have the following results (2015).
- The trivial transformation of r.c.c. corresponds to kerΦ. Then we gave a neat representative of basis of kerΦ[H1].
- The image ofφis a linear space. Then we gave a geometric generator of ImΦ. Further we studied the cokernel of Φ. Then we gave a method giving a representative of CokerΦ by using a graph obtained from given link diagram [H2].

6. We induced a $\mathbf{Z}_2$-linear map Ψ from the set based on the regions to the set based on the crossings defined by r.f.c.c. and made use of it to study r.f.c.c. The trivial transformation of r.c.c. corresponds to kerΨ (2016).
- We proved that dim(kerΦ)≦dim(kerΨ), and a necessary and sufficient condition for "<".
- We showed a necessary and sufficient condition that a given set of crossings is in ImΦ\ImΨ.

These give a generalization of a result of Inoue-Shimizu's for link diagram.

Reference
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