Variational problem on Hardy-Sobolev inequalities

(i) 3-dimensional problem

We know that the variational problem related to the Sobolev inequalities in the three dimensional case is special case, for instance, Brezis-Nirenberg (1983). The Hardy-Sobolev inequality has similar properties. In the three dimensional case, there are some open problems.

(ii) On best constant

We don't know the exact representation of the best constant in general. First, we study the best constant in a ball case. In addition, we study the relation between the best constant in the Dirichlet case and that in the Neumann case.

On noncompact sequences of Sobolev embeddings

(i) Embedding on whole space

It is known that the embedding from $W^{1,p}_{rad}(\mathbb{R}^N)$ to $L^p(\mathbb{R}^N)$ is noncompact because of the scaling saving the value of $L^p(\mathbb{R}^N)$-norm. But, we don't know that the noncompactness of the embedding $W^{1,p}_{rad}(\mathbb{R}^N) \hookrightarrow L^p(\mathbb{R}^N)$ is caused only by the scaling. We study to show this problem by using the profile decomposition.

(ii) Trudinger-Moser functional

The properties of noncompactness of Trudinger-Moser functional is different from that of the Sobolev embeddings. We study essentials of the noncompactness by studying variational problems, elliptic equations, and properties of solutions of equations.