## Research Plan

1. Let us denote by $U_m^{B\pm}(w)$ the set of signed unimodal factorizations of reduced words for an element $w$ of Coxeter group of type $B$. I gave the explicit algorithms for the odd Kashiwara operators on the element of $U_m^{B\pm}(w)$ [2]. However, the explicit algorithms for the even Kashiwara operators are still unknown. They are the type $B$ analogue of Kashiwara operators acting on the decreasing factorizations of reduced words of type $A$ introduced by Morse and Schilling [3]. While Edelman-Greene (EG) insertion plays a crucial role in [3], Kraśkiewicz insertion plays the role of EG insertion in [2]. Since the structure of Kraśkiewicz insertion is essentially the "doubled" EG insertion, it would be possible to construct the explicit algorithms for the even Kashiwara operators on the element of $U_m^{B\pm}(w)$ from Kashiwara operators in [3].

2. Let us denote by $\mathrm{PT}^{\pm}(\lambda/\mu)$ the set of skew signed primed tableaux of shape $\lambda/\mu$. The character of $\mathrm{PT}^{\pm}(\lambda/\mu)$ is the skew Schur $Q$-function so $\mathrm{PT}^{\pm}(\lambda/\mu)$ is an important combinatorial object. As an immediate consequence of the second part of Research Results, $\mathrm{PT}^{\pm}(\lambda/\mu)$ admits the $\mathfrak{q}(n)$-crystal structure. Kashiwara operators on $S \in \mathrm{PT}^{\pm}(\lambda/\mu)$ are expressed as the composition of Worley-Sagan (WS) sliding [4, 5], Kashiwara operators on the rectified shifted tableau, and the inverse of WS sliding. The second research plan is to find the explicit algorithms of Kashiwara operators on $S \in \mathrm{PT}^{\pm}(\lambda/\mu)$. The WS sliding and Kashiwara operators on a signed primed tableau intertwine each other in a complicated way, which makes the problem very difficult.

3. The research on the representation theory of generalized quantum groups will be started.

## References


[1] G. Hawkes, K. Paramonov, and A. Schilling, Crystal analysis of the type $C$ Stanley symmetric functions, *Electronic J. Combin.* **24** (2017) #P3.51.

[2] T. Hiroshima, $\mathfrak{q}$-crystal structure on primed tableaux and on signed unimodal factorizations of reduced words of type $B$, to appear in *Publ. RIMS Kyoto University*.

[3] J. Morse and A. Schilling, Crystal approach to affine Schubert calculus, *Int. Math Res. Not.* **8** (2016) 2239–2294.

[4] B. E. Sagan, Shifted tableaux, Schur Q-functions, and a conjecture of R. Stanley, *J. Combin. Theory Ser. A* **45** (1987) 62–103.

[5] D. R. Worley, A theory of shifted tableaux, Ph. D. thesis, MIT, 1984.